Note ♯7 (Exam2 Review)

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(1) Evaluate $\int \sqrt{5+4x-x^2}\, dx$.

(2) Evaluate $\int \frac{\sqrt{9x^2-4}}{x^4}\,dx$.

(3) Evaluate $\int \frac{1}{(x^2+4)^2}\,dx$.

(4) Evaluate $\int \frac{3x^2 + 4x + 3}{x^2 + 1}\, dx$.

(5) Evaluate $\int \frac{2x^3 + 11x^2 + 18}{x^2(x^2+9)}\, dx.$

(6) Evaluate $\displaystyle\int_0^{\infty} \frac{\sqrt{e^{-3x}+4}}{e^{3x}}\,dx$.

(7) Evaluate $\int_0^e \sqrt{x}\ln x \; dx$.

(8) Compute $\displaystyle\int_0^4 \frac{1}{(x-4)^4}\,dx$.

(9) Determine if the following sequences are increasing, decreasing, or not monotonic. Also, determine if each series is bounded.

(a) $a_n = 3 - e^{2n}$

(b) $a_n = \dfrac{(-1)^n(n+5)}{n^2+3n}$

(10) Determine whether the sequence converges or diverges. If it converges, what value does it converge to?

(a) $a_n = n\sin(\frac{5}{n})$

(b) $a_n = \dfrac{(-1)^{n-1}(7e^{2n} - 8)}{5n - 3e^{2n}}$

(11) Consider the recursive sequence $a_1 = 4$ and $a_{n+1} = \dfrac{5}{6 - a_n}$. Assuming the sequence is decreasing and bounded, determine if the sequence converges or diverges. If it converges, find its limit.

(12) For the series $\sum_{n=1}^{\infty} a_n$, the nth partial sum is given by $s_n = \dfrac{3-2n}{5n+1}$.

(a) Find a_n.

(b) Does the series converge or diverge? If it converges, find its sum.

(13) Determine if the series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{1}{n(3+\ln n)^3}$

(b) $\sum_{n=1}^{\infty} \frac{3}{n^2+4}$

(14) Determine if the series converges or diverges. If it converges, find its sum.

(a) $\sum_{n=1}^{\infty} \cos(\frac{1}{n})$

(b) $\sum_{n=5}^{\infty} 10(-\frac{2}{5})^{n-1}$

(c) $\sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{3n+1}}$

(d) $\sum_{n=1}^{\infty} [\cos(\frac{1}{n+3}) - \cos(\frac{1}{n+1})]$

(e) $\sum_{n=1}^{\infty} \frac{n^4+n^2}{5n-3n^4}$

(15) For what values of x does the series $\sum_{n=0}^{\infty} \frac{(x+3)^{n+1}}{5^n}$ converges? Find the sum of the series for those values of x.

(16) What is the smallest value of n that ensures s_n, the nth partial sum, approximates the sum of the series $\sum_{n=1}^{\infty} \frac{8}{(n+2)^3}$ with error less than 0.05?