Note ♯8 (The Comparison Test, Alternating Series, Absolute Convergence and the Ratio Test)

JD Kim

[The Comparison Test]

(1) Determine if the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n - 1}{5n + 4^n}$

(b) $\sum_{n=4}^{\infty} \frac{1}{\sqrt[3]{n^2-3}}$

(c) $\sum_{n=1}^{\infty} \frac{5+\sin n}{5n^3+n+1}$

(d) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{(2n+5)^4}$

[The Alternating Series Test]

(2) Determine if the series converges or diverges.

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{n+5}{n^2+3n}$

(b) $\sum_{n=1}^{\infty} (-1)^n e^{-n}$

(3) Use the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n+3)!}$ and the fact it converges by the Alternating Series Test for the following.

(a) Estimate the sum of the series by s_5.

(b) Find an upper bound for the error in the estimate.

(4) How many terms of the series do we need to add in order to find the sum so that the error less than 0.0005?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}$$

[**Absolute Convergence and the Ration Test**]

(5) Determine if the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty} \frac{\cos n}{n^4}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{\sqrt{n}}$

(c) $\sum_{n=1}^{\infty} \frac{(-2)^{2n+1} n^4}{3^{n-1}}$

(d) $\sum_{n=1}^{\infty} \frac{n^5}{(-10)^{n+1}}$

(e) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

