Note ♯9 (Power Series, Representations of Functions as Power Series)

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[Power Series]

(1) Find the interval and radius of convergence for the following power series.

(a) $\sum_{n=0}^{\infty} \frac{7^n x^n}{(2n+1)^3}$

(b) $\sum_{n=0}^{\infty} \frac{(2n)!(5x+1)^n}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{n(-5)^n(x+3)^{n+1}}{3n!}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{3(-1)^n(2x-5)^n}{4^{n+1}\ln n}$

(e) $\sum_{n=2}^{\infty} \frac{(2x-5)^n}{n4^n}$

[**Representations of Functions as Power Series**]

(2) Find a power series representation for the function. Determine the Radius of convergence.

(a) $f(x) = \dfrac{3x^3}{1-8x^3}$

(b) $f(x) = \dfrac{4x}{5 + 2x^2}$

(c) $f(x) = 3x^2 \ln(9 + 2x^3)$

(d) $f(x) = \dfrac{12x^5}{(4-x)^2}$

(3) Evaluate the indefinite integral as a power series. What is the radius of convergence?

(a) $\displaystyle\int \frac{x}{2x^6+3}\,dx$

(b) $\int x^2 \arctan(3x^2)\, dx$