



SERIES

Problem 1. What is a power series centered at a ? What is the *radius of convergence* and *interval of convergence* of a power series?

A power series centered at a is a series of the form $\sum_{k=0}^{\infty} c_k (x-a)^k$. There is an $0 \leq R \leq \infty$ such that this converges absolutely for $|x-a| < R$; diverges for $|x-a| > R$.

Problem 2. Write $f(x) = \frac{x}{1+x}$ as a power series. What is a ? What is c_k ?

Problem 3. Complete the theorem: For a given power series $\sum_{k=0}^{\infty} c_k(x - a)^k$ there are only three possibilities:

(1)

(2)

(3)

Problem 4. Find the ROC and IOC for $\sum_{k=0}^{\infty} \frac{(-3)^k x^k}{\sqrt{k+1}}$.

4

Problem 5. Find the ROC and IOC for $\sum_{k=0}^{\infty} \frac{k(x+2)^k}{3^{k+1}}$.

Problem 6. Find the ROC and IOC for $\sum_{k=0}^{\infty} (-1)^k kx^k$.

6

Problem 7. Find the ROC and IOC for $\sum_{k=0}^{\infty} (-1)^k a_k x^k$, where $a_0 = 1$ and $a_{k+1} = \frac{k}{k+1} a_k$.

Problem 8. Find the ROC and IOC for $\sum_{k=0}^{\infty} (-1)^k kx^k$.

8

Problem 9. Find the ROC and IOC for $\sum_{k=0}^{\infty} (-1)^k \frac{1}{k5^k} x^k$.

Problem 10. Find the ROC and IOC for $\sum_{k=0}^{\infty} \frac{(2x-1)^k}{5^k \sqrt{k}}$.

Problem 11. Assume that $\sum_{k=0}^{\infty} c_k 4^k$ converges. What can we say about:

(1) $\sum_{k=0}^{\infty} c_k (-2)^k$.

(2) $\sum_{k=0}^{\infty} c_k (-4)^k$.

Problem 12. Assume that $\sum_{k=0}^{\infty} c_k x^k$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of:

(1) $\sum_{k=0}^{\infty} c_k$

(2) $\sum_{k=0}^{\infty} c_k 8^k$

(3) $\sum_{k=0}^{\infty} c_k (-3)^k$

(4) $\sum_{k=0}^{\infty} (-1)^k c_k 9^k$.