Test Review

Problem 1. Determine whether the series converges or diverges: $\sum_{n=1}^{\infty} \frac{2ne^{-n}}{n^3+4n^2}$.

Problem 2. Determine whether the series converges or diverges: $\sum_{n=2}^{\infty} \frac{\sqrt{n}-1}{2+5\sqrt{n^3}}$.

Problem 3. Determine whether the series converges or diverges: $\sum_{n=1}^{\infty} \frac{(-5)^{n+1} n^4}{2^{2n}}$.

Problem 4. Determine whether the series converges or diverges: $\sum_{n=1}^{\infty} \frac{1}{n(3+\ln n)^3}$.

Problem 5.

For what values of x does the series $\sum_{n=0}^{\infty} \frac{(x+3)^{n+1}}{5^n}$ converge? Find the sum of the series for those values of x.

Problem 6. Determine whether the series converges absolutely, conditionally, or diverges: $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{4^{n+1}}$.

Problem 7. Determine whether the series converges absolutely, conditionally, or diverges: $\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{\sqrt{n}}$.

Problem 8. Determine whether the series converges absolutely, conditionally, or diverges: $\sum_{n=1}^{\infty} \frac{\ln(n)(-2)^{n+1}}{3 \cdot n!}$

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Problem 10. Find a power series representation of $f(x) = \dfrac{1}{1-5x}$ and determine the radius and interval of convergence.

$$\frac{1}{1-x} = \sum_{k=0}^{+\infty} x^k \qquad a = 0 \qquad (-1, 1) \qquad R = 1$$

$$\frac{1}{1-[5x]} = \sum_{k=0}^{+\infty} (-5x)^k = \sum_{k=0}^{+\infty} (-5)^k x^k$$

Converge: $|5x| < 1$ or $|x| < 1/5$.

So $R = 1/5$ IOC: $(-1/5, 1/5)$.

Problem 11. Find a power series representation of $f(x) = \dfrac{3}{2+x}$ and determine the radius and interval of convergence.

$$\frac{3}{2(1+\frac{x}{2})} = \frac{3}{2} \frac{1}{1-\left[\left(-\frac{x}{2}\right)\right]}$$

$$= \frac{3}{2} \sum_{k=0}^{+\infty} \left(-\frac{x}{2}\right)^k$$

$$= \frac{3}{2} \sum_{k=0}^{+\infty} \left(-\frac{1}{2}\right)^k x^k$$

$$= \sum_{k=0}^{+\infty} \frac{3}{2}\left(-\frac{1}{2}\right)^k x^k.$$

$\left|\frac{x}{2}\right| < 1 \iff |x| < 2.$

So IOC $= (-2, 2)$, ROC $= 2$.

Problem 12. Find a power series representation of $f(x) = \dfrac{x^4}{8 - x^3}$ and determine the radius and interval of convergence.

$$\frac{x^4}{8\left(1 - \frac{x^3}{8}\right)} = \frac{x^4}{8} \; \frac{1}{1 - \frac{x^3}{8}}$$

$$= \frac{x^4}{}$$

Problem 13. Find a power series representation of $f(x) = \dfrac{x^4}{8 - x^3}$ and determine the radius and interval of convergence.

Problem 14. If $f(x) = \sum_{n=0}^{\infty} \frac{n^4(8x)^n}{n!}$, find $f'(x)$ and $\int f(x)\,dx$.

Problem 15. Find a power series representation of $\dfrac{1}{(5+2x)^2}$ and determine the radius and interval of convergence.

Problem 16. Find a power series representation of $\dfrac{1}{(5+2x)^2}$ and determine the radius and interval of convergence.

Problem 17. Find a power series representation of $\dfrac{x^2}{(5+2x)^2}$ and determine the radius and interval of convergence.

Problem 18. Find a power series representation of $x\ln(2-x^2)$ and determine the radius and interval of convergence.

Problem 19. Find a power series representation of $\int x\ln(2-x^2)\,dx$ and determine the radius and interval of convergence.

Problem 20. Express $\int_{x=0}^{1} e^{x^2} dx$ as a series.

Problem 21. Find $f^{(50)}(2)$ if $f(x) = \sum_{n=0}^{\infty} \frac{10^{n+1}(x-2)^n}{(n+5)!}$, that is the 50^{th} derivative of f at $x = 2$.

Problem 22. Find the Taylor Series centered at $a = 4$ if $f(x) = \dfrac{1}{x^2}$.

Problem 23. Find the Maclaurin Series for $f(x) = e^{-x^2}$.

Problem 24. Find the Maclaurin Series for $f(x) = x^2 \sin\left(\frac{x^4}{4}\right)$

Problem 25. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n (\pi)^{2n}}{4^{2n}(2n)!}$.

Problem 26. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n(\pi)^{2n}}{4^{2n}(2n)!}$.

Problem 27. Assume that $\sum_{k=0}^{\infty} c_k 4^k$ converges. What can we say about:

(1) $\sum_{k=0}^{\infty} c_k(-2)^k$.

(2) $\sum_{k=0}^{\infty} c_k(-4)^k$.