Test Review

Problem 1. Sketch the curve $x = 1 - t^2$, $y = 2t - t^2$ with $-1 \leq t \leq 2$.

↓ Eliminate the parameter

$t^2 = 1 - x$

$t = \sqrt{1-x}$

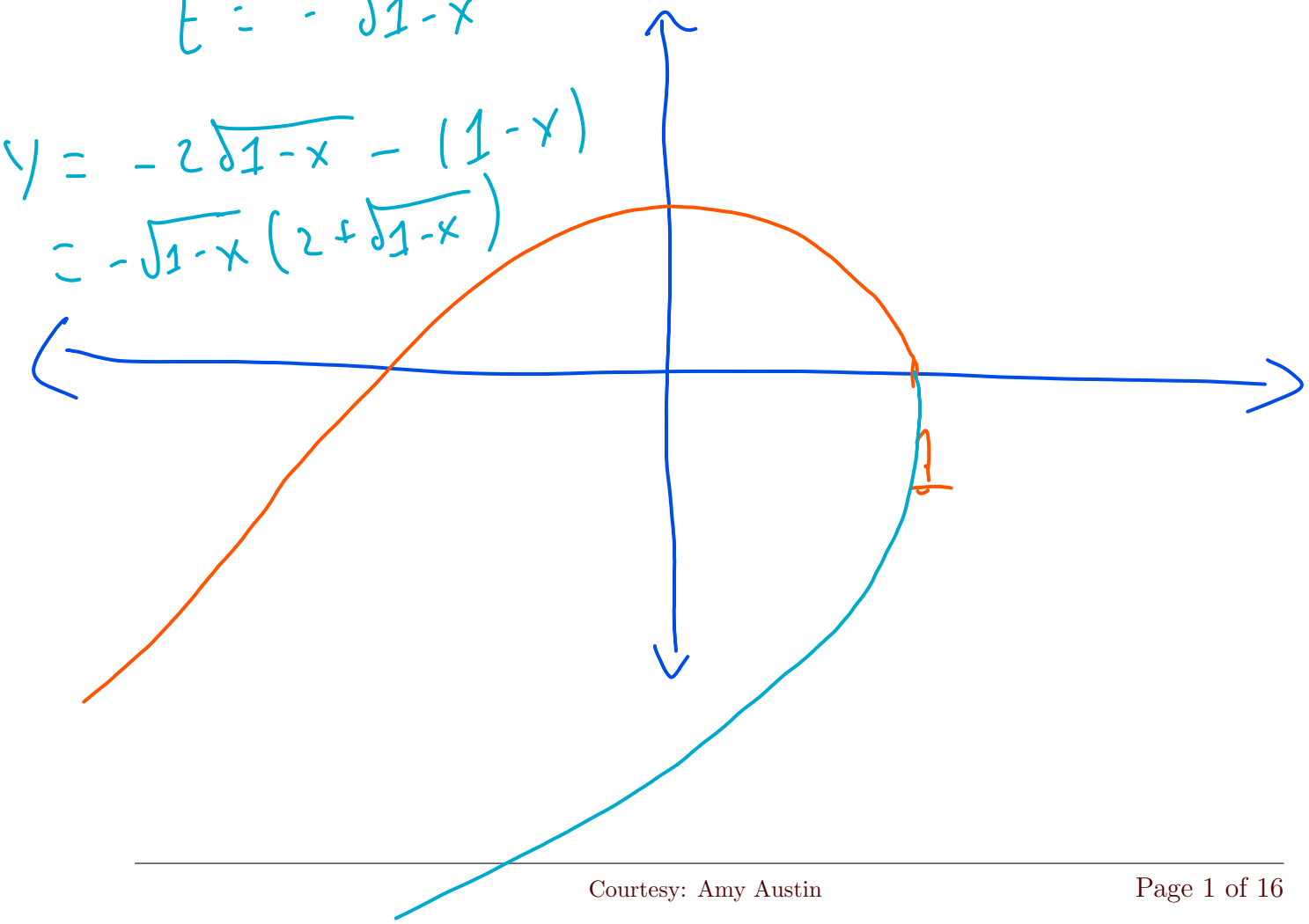
$y = 2\sqrt{1-x} - (1-x)$

$= \sqrt{1-x}\left[2 - \sqrt{1-x}\right]$

$t = -\sqrt{1-x}$

$y = -2\sqrt{1-x} - (1-x)$

$= -\sqrt{1-x}\left(2 + \sqrt{1-x}\right)$



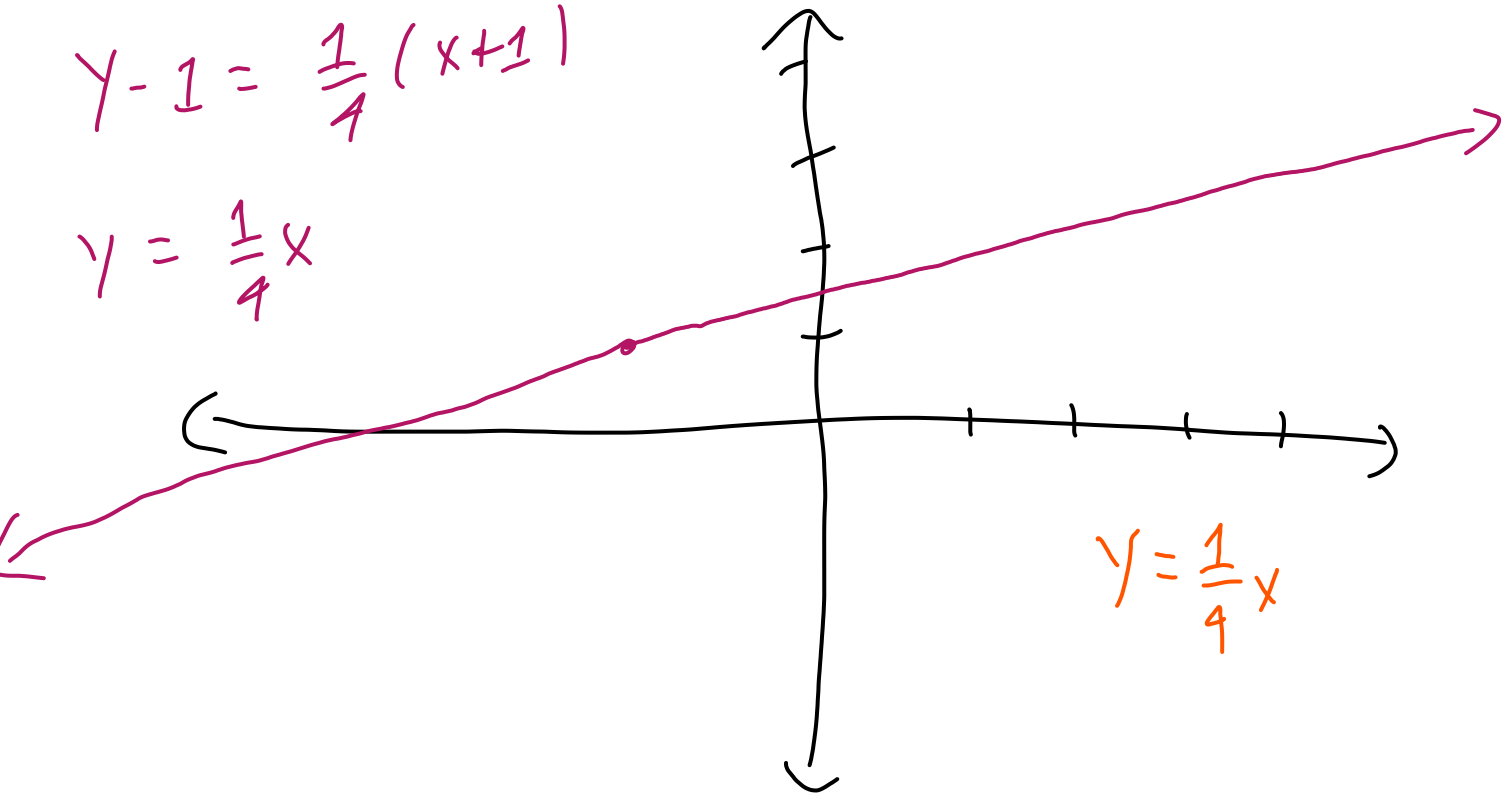
Problem 2. Sketch $x = 2t - 1$, $y = \frac{1}{2}t + 1$.

$\langle 2t-1, \frac{1}{2}t+1 \rangle = \langle -1, 1 \rangle + t\langle 2, \frac{1}{2} \rangle$

line thru origin $m = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x+1)$

$y = \frac{1}{4}x$



$y = \frac{1}{4}x$

Problem 3. Sketch $x = \sin t$, $y = 1 - \cos t$, $0 \leq t, \leq 2\pi$.

Problem 4. Sketch $x = \sin \frac{1}{2}t$ $y = \cos \frac{1}{2}t, -\pi \leq t \leq \pi$.

Problem 5. Sketch $x = e^t$, $y = e^{-2t}$.

Problem 6. Sketch $x = t^2$, $y = \ln t$.

Problem 7. Find parametric equations for $x^2/a^2 + y^2/b^2 = 1$.

Problem 8. Find the equation of the tangent line to the curve $x = t^3 + 1$, $y = t^4 + t$ at $(0, 2)$.

Problem 9. Find the equation of the tangent line to the curve $x = t\cos t$, $y = t\sin t$ at $(\pi, 0)$.

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Problem 10. Find the equation of the tangent line to the curve $x = t^3 + 1$, $y = t^4 + t$ at $(0, 2)$.

Problem 11. Let $x = t^2 + 1$ and $y = t^2 + t$. Find $\frac{dy}{dx}$ and $\frac{dy^2}{dx^2}$. For what values of t is the curve CD?

Problem 12. Let $x = t - \ln t$ and $y = t + \ln t$. Find $\frac{dy}{dx}$ and $\frac{dy^2}{dx^2}$. For what values of t is the curve CD?

Problem 13. Show that $x = \cos t$, $y = \sin t \cos t$ has two tangents at $(0, 0)$. Find their equations.

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Problem 14. Find the length of $x = t + e^{-t}$ and $y = t - e^{-t}$ for $0 \leq t \leq 2$.

Problem 15. Find the lentsh of $x = t - 2\sin t$, $y = 1 - 2\cos t$, $0 \leq t \leq 4\pi$.

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Problem 16. Find the lentsh of $x = e^t \cos t$, $y = e^t \sin t$, $0 \leq t \leq \pi$.