Test 1 Review

Problem 1. \[ \int \frac{\cos^3(\ln x)}{x} \, dx \]
Problem 2. The force required to hold a spring stretched to a length of 7 m is 5 N. Find the work required to stretch the spring from a length of 4 m to 8 m. The natural length of the spring is 3 m.
Problem 3.
Find the volume of the solid $S$ whose base is bounded by the region $x^2 + 4y^2 = 4$, and cross-sections perpendicular to the $y$-axis are isosceles triangles with height equal to the base.
Problem 4. Find the area bounded by \( x = 3y - y^2 \) and \( y = -\frac{x}{2} \).
Problem 5. \( \int \frac{\ln x}{\sqrt{x}} \, dx \)
Problem 6. The region bounded by $y = \frac{1}{x^2}$, $x = 1$, $x = e$, and $y = 0$ is rotated around the $y$-axis. Find the volume.
Problem 7. The region bounded by $x + y^2 = 4$ and $x - y = 2$ is rotated around the line $x = -1$. Set up but do not evaluate an integral representing the volume of the solid.
Problem 8. $\int_{0}^{2} x^2 e^{3x} \, dx$
Problem 9. Find the area bounded by $y = 7 - x^2$ and $y = 2x^2 - 5$. 
Problem 10. Set up but do not evaluate an integral for the volume of the solid obtained by rotating the region bounded by $y = x^2 - x$ and $y = 2$ rotated around the line $x = 3$. 
Problem 11. \[ \int \frac{x^3}{(x^2 + 1)^8} \, dx \]
Problem 12. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$ around the $y$-axis.
Problem 13. \[ \int \tan^6 x \sec^4 dx \]
Problem 14. \( \int_0^{\pi/6} \sin^2(5x) \, dx \)
Problem 15. Find $\int e^x \sin(8x) \, dx$
Problem 16. A bucket attached to a 20 pound rope is used to draw water out of an 80 ft well. The bucket weighs 1 pound and holds 26 pounds of water. How much work is done in drawing up one full bucket of water?
Problem 17. Consider the region $R$ bounded by $y = x^3$, $y = 8$, and $x = 0$. Suppose a tank is in the shape of the region $R$ revolved around the $y$-axis, and the units are measured in meters. If the tank is filled with water to a depth of 3 m, set up but do not evaluate an integral that gives the work done in pumping all the water out of a 1 m high spout. Use $\rho g$ for the weight density of water.
Continue work here.