Week in Review Session ♯10 (Sections 5.1-5.3)

1. Determine the radius of convergence for the following power series:
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}$

2. For the equation $(x^2+1)y'' + xy' - y = 0$
   (a) Determine a lower bound for the radius of convergence of the series solutions of the differential equation about $x_0 = 0$.

   (b) Seek its power series solution about $x_0 = 0$; find the recurrence relation.

   (c) Find the general term of each solution $y_1(x)$ and $y_2(x)$.

   (d) Find the first four terms in each of two solutions y_1 and y_2. Show that $W[y_1, y_2](0) \neq 0$.

3. For the equation $(x^2+1)y'' - 6y = 0$
   (a) Determine a lower bound for the radius of convergence of the series solutions of the differential equation about $x_0 = 0$.

   (b) Seek its power series solution about $x_0 = 0$; find the recurrence relation.

(c) Find the general term of each solution $y_1(x)$ and $y_2(x)$.

(d) Find the first four terms in each of two solutions y_1 and y_2. Show that $W[y_1, y_2](0) \neq 0$.

4. Seek a power series solution for the initial value problem

$$y'' - (1 + x)y = 0, \quad y(0) = 1, y'(0) = -2$$

up to the terms of degree 5. Then do the same for finding the general solution of the equation.

5. For the following equation, determine $\phi''(x_0)$ and $\phi'''(x_0)$, for the given point x_0 if $y = \phi(x)$ is a solution of the given initial-value problem.

$$y'' + x^2y' + (\sin x)y = 0; \quad y(0) = a_0, \quad y'(0) = a_1$$