Week in Review Session ♯11 (Sections 7.1-7.9)

1. Transform the given initial value problem into an initial value problem for three first-order equations, and write it in matrix form.
$$u''' + p(t)u'' + q(t)u' + r(t)u = g(t), \quad u(0) = u_0, \quad u'(0) = u_1, \quad u''(0) = u_2$$

2. Given the two vectors
$$\mathbf{x_1}(\mathbf{t}) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \mathbf{e^t} \text{ and } \mathbf{x_2}(\mathbf{t}) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \mathbf{e^{2t}},$$
find a differential equation $\mathbf{x'} = \mathrm{A}\mathbf{x}$ for which $\mathbf{x_1}(\mathbf{t})$ and $\mathbf{x_2}(\mathbf{t})$ are solutions.

3. Find the general solution of the system and the fundamental matrix. Classify the type of the critical point (0,0), and determine whether it is stable or unstable. Sketch the phase portrait.

   (a) $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$

   (b) $\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$

   (c) $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$

4. Classify the type and stability of the equilibrium point(s) of the system
$$\mathbf{x}' = \begin{pmatrix} \alpha - 1 & \alpha + 1 \\ -2/3 & 1/3 \end{pmatrix} \mathbf{x}$$
for different values of the parameter α.

5. Find the general solution of the nonhomogeneous system.
$$\mathbf{x}' = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t^{-1} \\ 2t^{-1} + 4 \end{pmatrix}, \qquad t > 0$$

