Week in Review Session ♯12 Final Exam Review

Review for Final Exam (Please note that this review may not cover all sections of your Final Exam)

1. (Chapter 2) Find the general solution of the given differential equation.
   (a) $2\sqrt{x}\,y' = \sqrt{1-y^2}$

   (b) $ty' + y = 3t\cos t, \qquad t > 0.$

2. (Chapter 2) Given the differential equation

$$y'(t) = y^3 - 2y^2 + y$$

   (a) Find the equilibrium solutions.
   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, or semistable.
   (c) Sketch the graph of some solutions.
   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, determine the behavior of $y(t)$ as t increases.
   (e) Do any solutions 'blow up in finite time,' namely, do they admit a vertical asymptote?

3. (Chapter 2) Solve $y' = \dfrac{(1+x)e^x}{xe^x - ye^y}$.

4. (Chapter 3) Find the solution to the given initial value problem.

   $y'' - 4y' + 3y = 0, \quad y(0) = 3, \quad y'(0) = 4.$

5. (Chapter 3) If the Wronskian of f and g is $t\cos(t) - \sin(t)$, and if $u = 2f - 3g$, and $v = f + g$, find the Wronskian of u and v.

6. (Chapter 3) Find the general solution of the equation $y'' + 2y' + y = 4e^{-t}$.

7. (Chapter 3) Solve the differential equation

$$t^2y'' + 2ty' - 2y = t, \quad t > 0$$

assuming $y_1(t) = t$ is a solution to the corresponding homogeneous equation.

8. (Chapter 5) For the equation $(x^2 + 1)y'' - 6y = 0$
   (a) Determine a lower bound for the radius of convergence of the series solutions of the differential equation about $x_0 = 0$.

   (b) Seek its power series solution about $x_0 = 0$; find the recurrence relation.

   (c) Find the general term of each solution $y_1(x)$ and $y_2(x)$.

   (d) Find the first four terms in each of two solutions y_1 and y_2. Show that $W[y_1, y_2](0) \neq 0$.

9. (Chapter 5) For the following equation, determine $\phi''(x_0)$ and $\phi'''(x_0)$, for the given point x_0 if $y = \phi(x)$ is a solution of the given initial-value problem.

$$y'' + x^2y' + (\sin x)y = 0; \quad y(0) = a_0, \quad y'(0) = a_1$$

10. (Chapter 6) Find the solution of the initial value problem
   (a) $y'' + y = \delta(t - 2\pi)\cos t, \qquad y(0) = 0, \quad y'(0) = 1.$

   (b) $y'' + 3y' + 2y = \begin{cases} 1, & 0 \leq t < 10 \\ 0, & 10 \leq t \end{cases}, \qquad y(0) = 0, \quad y'(0) = 0.$

11. (Chapter 6) Solve the initial value problem $y'' - 2y' - 3y = g(t),\ y(0) = 1,\ y'(0) = -3.$

12. (Chapter 7) Find the general solution of the system and the fundamental matrix. Classify the type of the critical point (0,0), and determine whether it is stable or unstable. Sketch the phase portrait.

(a) $\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -5 & -1 \end{pmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$

(d) $\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$

(e) $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$