Week in Review Session ♯1 (Sections 1.1-1.3)

This document contains the answers and video solutions to the posed problems. Click the red box to watch the video solution. You can also watch all videos by viewing the Session 1 playlist. In the event that this weekly review handouts is updated, the Session video playlist will reflect the most updated problem set. Closed captions are available for all videos and the speed of the videos may be adjusted inside of "Settings" or the cog in the bottom right corner.

1. Given the differential equation $\frac{dy}{dt} = ty - 1$.

(a) What is the slope of the graph of the solutions at $(0, 1)$, at the point $(1, 1)$, at the point $(3, -1)$, at the point $(0, 0)$?

Answer:
$(t, y) = (0, 1),\ m_1 = -1$
$(t, y) = (1, 1),\ m_2 = 0$
$(t, y) = (3, -1),\ m_3 = -4$
$(t, y) = (0, 0),\ m_4 = -1$

Click here to see video solution to problem #1(a)

(b) Find all the points where the tangents to the solution curves are horizontal.

Answer: $\{(t, \frac{1}{t}) \mid t \neq 0\}$

Click here to see video solution to problem #1(b)

(c) Describe the nature of the critical points.

Answer: At the critical points, $y'' = y$. If $y > 0$ then $y'' = y > 0$ so we have a local minimum at critical points. If $y < 0$ then $y'' = y < 0$ so we have a local maximum at critical points.

Click here to see video solution to problem #1(c)

2. The instantaneous rate of change of the temperature T of coffee at time t is proportional to the difference between the temperature M of the air and the temperature T at time t.
   (a) Find the mathematical model for the problem.

   Answer: $\dfrac{dT}{dt} = -k(T - M) \quad k > 0$

   Click here to see video solution to problem #2(a)

   (b) Given that the room temperature is 75° and $k = 0.08$, find the solutions to the differential equation.

   Answer: $T(t) = 75 + Ce^{-0.08t}$

   Click here to see video solution to problem #2(b)

   (c) The initial temperature of the coffee is 200°F. Find the solution to the problem.

   Answer: $T(t) = 75 + 125e^{-0.08t}$

   Click here to see video solution to problem #2(c)

3. Your swimming pool containing 60,000 gal of water has been contaminated by 5 kg of a non toxic dye that leaves a swimmer's skin an unattractive green. The pool's filtering system can take water from the pool, remove the dye, and return the water to the pool at a flow rate of 200 gal/min.
   (a) Write down the initial value problem for the filtering process; let $q(t)$ be the amount of dye in the pool at any time t.

   Answer:

$$\begin{cases} \dfrac{dq}{dt} = -\dfrac{1}{300}q(t) \\ \\ q(0) = 5 \end{cases}$$

   Click here to see video solution to problem #3(a)

   (b) Solve the problem.

   Answer: $q(t) = 5e^{-\frac{1}{300}t}$

   Click here to see video solution to problem #3(b)

(c) You have invited several dozen friends to a pool party that is scheduled to begin in 4 hours. You have also determined that the effect of the dye is imperceptible if its concentration is less than 0.02 g/gal. Is your filtering system capable of reducing the dye concentration to this level within 4 hours?

Answer: Four hours is not enough time.

Click here to see video solution to problem #3(c)

4. Given the following differential equations, classify each as an ordinary differential equation, partial differential equation, give the order. If the equation is an ordinary differential equation, say whether the equation is linear or non linear.

(a) $\frac{dy}{dx} = 3y + x^2$.

(b) $5\frac{d^4y}{dx^4} + y = x(x-1)$.

(c) $\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r} + kN$.

(d) $\frac{dx}{dt} = x^2 - t$.

(e) $(1+y^2)y'' + ty' + y = e^t$.

Answer:

Part	ODE	PDE	Order	Linear/Non Linear
(a)	✓		1	Linear
(b)	✓		4	Linear
(c)		✓	2	Linear
(d)	✓		1	Non Linear
(e)	✓		2	Non Linear

Click here to see video solution to problem #4

5. (a) Show that $f(x) = (x^2 + Ax + B)e^{-x}$ is solution to

$$y'' + 2y' + y = 2e^{-x}$$

for all real numbers A and B.

Answer: For verification that $f(x)$ is a solution for part (a), see video below.

Click here to see video solution to problem #5(a)

(b) Find a solution that satisfies the initial condition $y(0) = 3$ and $y'(0) = 1$.

Answer: $y = (x^2 + 4x + 3)e^{-x}$

Click here to see video solution to problem #5(b)

6. Determine for which values of r the function t^r is a solution of the differential equation

$$t^2y'' - 4ty' + 4y = 0, \quad t > 0.$$

Answer: $r = 1, 4$

Click here to see video solution to problem #6

7. For which values of r is the function $(x-1)e^{-rx}$ solution to $y'' - 6y' + 9y = 0$?

Answer: $r = -3$

Click here to see video solution to problem #7

Video errata: At the 9:46 mark, there is a sign error, we should have $y = (x-1)e^{3x}$.

8. In curling, the player has to slide a stone of mass m on a smooth surface with friction coefficient μ. If the air friction is proportional to the velocity of the stone with a drag coefficient of γ, find the stopping time and the stopping distance of the stone if the initial velocity is v_0.

Answer: The stopping time of the stone is $T = \frac{m}{\gamma}\ln(1 + \frac{\gamma v_0}{\mu m g})$ and the stopping distance of the stone is $D = \frac{m}{\gamma}\left(\frac{\mu m g}{\gamma} + v_0\right)\left[1 - \frac{\mu m g}{\mu m g + \gamma v_0}\right] - \mu g(\frac{m}{\gamma})^2 \ln(1 + \frac{\gamma v_0}{\mu m g})$

Click here to see video solution to problem #8