Week in Review Session ♯2 (Sections 2.1-2.3)

This document contains the answers and video solutions to the posed problems. Click the red box to watch the video solution. You can also watch all videos by viewing the Session 2 playlist. In the event that this weekly review handouts is updated, the Session video playlist will reflect the most updated problem set. Closed captions are available for all videos and the speed of the videos may be adjusted inside of "Settings" or the cog in the bottom right corner.

1. Find the general solution of the given differential equation.

   (a) $y' + 2ty = 2te^{-t^2}$

   Answer: $y = (t^2 + C)e^{-t^2}$

   Click here to see video solution to problem #1(a)

   (b) $2\sqrt{x}\,y' = \sqrt{1-y^2}$

   Answer: $y = \sin(\sqrt{x} + C)$, equilibrium solutions at $y = \pm 1$

   Click here to see video solution to problem #1(b)

   (c) $ty' + y = 3t\cos t, \qquad t > 0.$

   Answer: $y = \dfrac{3(t\sin(t) + \cos(t)) + C}{t}$

   Click here to see video solution to problem #1(c)

2. Find the solution to the initial value problem and the interval of validity in each case.

   (a) $2\sqrt{x}\dfrac{dy}{dx} = \cos^2 y, \qquad y(4) = \dfrac{\pi}{4}.$

   Answer: $y = \arctan(\sqrt{x} - 1)$, $I.V. = (0, \infty)$

   Click here to see video solution to problem #2(a)

   (b) $\dfrac{dy}{dt} + \dfrac{2y}{t} = \dfrac{\cos t}{t^2} \qquad y(1) = \dfrac{1}{2}, \qquad t > 0.$

   Answer: $y = \dfrac{\sin(t) + \frac{1}{2} - \sin(1)}{t^2}$, $I.V. = (0, \infty)$

   Click here to see video solution to problem #2(b)

3. Consider the initial value problem

$$y' + 2y = 5 - t, \qquad y(0) = y_0$$

Find the value y_0 for which the solution touches, but does not cross the t-axis.

Answer: $y_0 = \frac{11}{4} - \frac{1}{4}e^{10}$

Click here to see video solution to problem #3

Video errata: At the 10:30 mark, I wrote $\frac{11}{4} - \frac{t_1}{4} + (y_0 - \frac{11}{4})e^{-2t_1} = 0$ but we should have $\frac{11}{4} - \frac{t_1}{2} + (y_0 - \frac{11}{4})e^{-2t_1} = 0$. Continuing from there, we still get that $t_1 = 5$, but now we have the equation $\frac{11}{4} - \frac{5}{2} + (y_0 - \frac{11}{4})e^{-10} = 0$. Solving for y_0 then gives us that $y_0 = \frac{11}{4} - \frac{1}{4}e^{10}$.

4. A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gallons of water. Brine containing 2 lb/gal of salt flows into the tank at a rate of 4 gal/min, and the well-stirred mixture flows out of the tank at a rate of 3 gal/min. How much salt does the tank contain when it is full?

Answer: 202 lbs of salt

Click here to see video solution to problem #4

5. In a certain culture of bacteria, the number of bacteria increases sixfold in 10hrs. How long does it take for the population to double?

Answer: Approximately 3.87 hours

Click here to see video solution to problem #5

6. Consider a large lake formed by damming a river that initially holds 200 million liters of water. Because a nearby chemical plant uses the lake's water to clean its reservoirs, 1,000 liters of brine, each containing $100(1 + \cos t)$ kilograms of dissolved pollution, run into the lake every hour. Let's make the simplifying assumption that the mixture, kept uniform by stirring, runs out at the rate of 1,000 liters per hour and no additional spraying causes the lake to become even more contaminated. Find the amount of pollution $p(t)$ in the lake at any time t, and determine its variations in the long run.

Answer: $p(t) = 2 \cdot 10^7 + 100 \cdot 5 \cdot 10^{-6}\cos(t) + 100\sin(t) - 2 \cdot 10^7 e^{-5 \cdot 10^{-6} t}$, as t gets larger, $p(t) \approx 2 \cdot 10^7 + 100\sin(t)$.

Click here to see video solution to problem #6

7. A cake is removed from an oven at 210° F and left to cool at room temperature, which is 70° F. After 30 min the temperature of the cake is 140° F. When will it be 100° F?

   Answer: Approximately 66 minutes and 40 seconds.

   Click here to see video solution to problem #7

8. A ball with mass 1kg is thrown upward with initial velocity 20 m/s from the roof of a building 50 m high. A force due to the resistance of the air of $v/10$, where the velocity is measured in m/s, acts on the ball. Find the maximum height above the ground that the ball reaches.

   Answer: Approximately 68 m.

   Click here to see video solution to problem #8

   Video errata: At the 40:33 mark, the answer should be approximately equal to 68 m. This is also noted in the video.

9. In a typical Swedish summer day, temperature fluctuates sinusoidally between 10 (at midnight) and 30 (at noon) degrees Celsius. Assume at a party someone forgets a root beer with a temperature of 5 degrees at 10 pm, but finds it again at 2 am. What will be the temperature of the beer then, assuming a heat transfer coefficient of 0.1 per hour.

   Answer: Approximately $9.1°C$.

   Click here to see video solution to problem #9