Week in Review Session ♯3 (Sections 2.4-2.6)

1. Determine an interval in which the solution of the following initial value problem is certain to exist.
$$(t^2-1)y' + (\sin t)y = \frac{\cot t}{t^2-4t+3}, \qquad y(2) = -1$$

2. State where in the ty-plane the hypothesis of the Existence and Uniqueness theorem are satisfied for the following differential equations.

   (a) $y' = \dfrac{\ln(ty)}{1-(t^2+y^2)}$

   (b) $y' = (t^2-y)^{1/3}$.

3. Solve the following initial value problems and determine how the interval in which the solution exists depends on the initial value y_0.

   (a) $y' = \dfrac{-4}{t}y, \qquad y(2) = y_0$

   (b) $y' + y^3 = 0 \qquad y(t_0) = y_0$

4. Verify that both $y_1 = 1 - t$ and $y_2 = \dfrac{-t^2}{4}$ are solutions to the same initial value problem

$$y'(t) = \frac{-t + \sqrt{t^2 + 4y}}{2}, \qquad y(2) = -1.$$

Does it contradict the existence and uniqueness theorem?

5. Given the differential equation

$$y' = y^3 - 4y$$

(a) Find the equilibrium solutions.
(b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.
(c) Graph some solutions.
(d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when t increases.

6. Given the differential equation

$$y'(t) = y^3 - 2y^2 + y$$

(a) Find the equilibrium solutions.
(b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, or semistable.
(c) Sketch the graph of some solutions.
(d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, determine the behavior of $y(t)$ as t increases.
(e) Do any solutions 'blow up in finite time,' namely, do they admit a vertical asymptote?

7. Determine if the equations are exact and solve the ones that are:
(a) $(2x + 5y)dx + (5x - 6y)dy = 0.$

(b) $1 + \dfrac{y}{x} - \dfrac{1}{x}y' = 0$

(c) $(\sin(2t) + 2y)dy + (2y\cos(2t) - 6t^2)dt = 0$

8. Show that the following equations are not exact. However, if you multiply by the given integrating factor, the resulting equation is exact.
   (a) $(x^2 + y^2 - x)dx - ydy = 0 \qquad \mu(x, y) = \dfrac{1}{x^2 + y^2}$

   (b) $3(y + 1)dx - 2xdy = 0, \qquad \mu(x, y) = \dfrac{y + 1}{x^4}$

9. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

   and then solve the equation.

10. Solve the IVP $(3x^2 + 2xy^2)dx + 2x^2ydy = 0,\ y(2) = -3$.

11. Solve $(x^4 \ln x - 2xy^3)dx + 3x^2y^2dy = 0$.

12. Solve $y' = \dfrac{(1 + x)e^x}{xe^x - ye^y}$.

13. Solve

$$ydx = (y^2 + x^2 + x)dy,$$

   if we know there is an integrating factor of the form $\mu = \phi(x^2 + y^2)$.