Week in Review Session ♯4 (Sections 3.1-3.3)

This document contains the answers and video solutions to the posed problems. Click the red box to watch the video solution. You can also watch all videos by viewing the Session 4 playlist. In the event that this weekly review handout is updated, the Session video playlist will reflect the most updated problem set. Closed captions are available for all videos and the speed of the videos may be adjusted inside of "Settings" or the cog in the bottom right corner.

1. Verify that the functions y_1 and y_2 are solutions of the given differential equation. Do they constitute a fundamental set of solutions?

$$x^2y'' - x(x+2)y' + (x+2)y = 0, \qquad x > 0 \quad y_1(x) = x, \quad y_2(x) = xe^x.$$

Answer: The functions y_1 and y_2 constitute a fundamental set of solutions. For verification that y_1 and y_2 are solutions of the given differential equation, see video below.

Click here to see video solution to problem #1

2. If the Wronskian of f and g is $3e^{4t}$ and $f(t) = e^{2t}$, find $g(t)$.

Answer: $g(t) = (3t + C)e^{2t}$

Click here to see video solution to problem #2

3. Solve the initial value problem

$$y'' - y' - 2y = 0, \qquad y(0) = \alpha, \quad y'(0) = 2.$$

Find α so that the solution approaches zero as $t \to \infty$.

Answer: $y = \frac{1}{3}(\alpha+2)e^{2t} + \frac{2}{3}(\alpha-1)e^{-t}$, if $\alpha = -2 \implies y = -2e^{-t} \to 0$ as $t \to \infty$

Click here to see video solution to problem #3

4. Find the solution to the given initial value problem.

$$y'' - 2y' + 5y = 0 \qquad y(\pi/2) = 0, \quad y'(\pi/2) = 2.$$

Answer: $y = -e^t(e^{-\frac{\pi}{2}})\sin(2t)$

Click here to see video solution to problem #4

5. Solve the initial value problem

$$t^2y'' + 3ty' - 3y = 0, \qquad y(1) = 3, \quad y'(1) = 3/2$$

given that $y_1(t) = t$ is a solution of the equation.

Answer: $y = \dfrac{21}{8}t + \dfrac{3}{8}t^{-3}$

Click here to see video solution to problem #5

6. If the differential equation

$$ty'' + 2y' + te^t y = 0$$

has a fundamental set of solutions y_1 and y_2 and $W(y_1, y_2)(1) = 2$, find the value of $W(y_1, y_2)(5)$.

Answer: $W(y_1, y_2)(5) = \dfrac{2}{25}$

Click here to see video solution to problem #6