# Week in Review Session ♯6 (Sections 3.7-3.8) & Exam 1 Review

1. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in, then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position $u$ of the mass at any time $t$. Determine the frequency, period, amplitude and phase angle of the motion.

2. A spring is stretched 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass 5 m/s. If the mass is pulled down 5 cm below its equilibrium position and given an initial velocity of 10 cm/s, determine its position $u$ at any time. Find the quasifrequency of the motion.

3. A spring is stretched 6 in by a mass that weighs 8 lb. The mass is attached to a dashpot mechanism that has a damping constant of 0.25 lb·s/ft and is acted by an external force of $4\cos 2t$ lb.
   (a) Find the steady-state response of this system.

   (b) If the given mass is replaced by a mass $m$, determine the value of $m$ for which the amplitude of the steady-state response is maximum.

   (c) If the mass is the same as in the problem, determine the value $\omega$ of the frequency of the external force $4\cos \omega t$ lb at which "*practical resonance*" occurs, i.e., the amplitude of the steady-state response is maximized.

4. A mass weighing 4 lb stretches a spring 1.5 in. The mass is given a positive displacement 2 in from its equilibrium position and released with no initial velocity. Assuming that there is no damping and the mass is acted on by an external force of $2\cos 3t$ lb,
   (a) Formulate the initial value problem describing the motion of mass

(b) Solve the initial value problem.

(c) If the given external force is replaced by a force $4\cos\omega t$ of frequency $\omega$, find the value of $\omega$ for which resonance occurs.

5. A 3 kg object is attached to a spring and will stretch the spring 392 mm by itself. There is no damping in the system and a forcing function of the form $F(t) = 10\cos(\omega t)$ is attached to the object and the system will experience resonance. If the object is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward find the displacement at any time $t$.

**Review for Exam 1**

6. For the initial value problem $(t^2-4)y' + 2ty = 3t^2, \quad y(1) = -3$
   (a) Determine an interval in which the solution to the initial value problem is certain to exist.

   (b) Solve the initial value problem.

7. A large tank initially contains 10 L of fresh water. A brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 2 L/min. Determine the concentration of salt in the tank as a function of time.

8. Given the differential equation
$$\frac{dy}{dt} = 7y - y^2 - 10$$
   (a) Find the equilibrium solutions.
   (b) Sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable.
   (c) Sketch the graph of some solutions.
   (d) Determine the behavior of $y(t)$ as $t$ increases for all possible values of $y(0) = y_0$.
   (e) Do any solutions admit a vertical asymptote?
   (f) Solve the equation.

9. Find an integrating factor for the equation

$$(y^2 + xy) + (x^2 + 3xy)y' = 0$$

and then solve the equation.