Week in Review Session ♯7 (Sections 6.1-6.3)

This document contains the answers and video solutions to the posed problems. Click the red box to watch the video solution. You can also watch all videos by viewing the Session 7 playlist. In the event that this weekly review handout is updated, the Session video playlist will reflect the most updated problem set. Closed captions are available for all videos and the speed of the videos may be adjusted inside of "Settings" or the cog in the bottom right corner.

1. Use the definition to find the Laplace transforms of

   (a) $f(t) = e^{at}$ where a is a non zero real number.

   Answer: $\dfrac{1}{s-a}$ provided $s > a$

   Click here to see video solution to problem #1(a)

   (b) $f(t) = \sin(bt)$ where b is a non zero real number.

   Answer: $\dfrac{b}{s^2+b^2}$ provided $s > 0$

   Click here to see video solution to problem #1(b)

   (c) $f(t) = \begin{cases} 5-t & 0 \leqslant t < 2 \\ 3t & 2 \leqslant t. \end{cases}$

   Answer: $\dfrac{5}{s} - \dfrac{1}{s^2} + e^{-2s}\left(\dfrac{3}{s} + \dfrac{4}{s^2}\right)$ provided $s > 0$

   Click here to see video solution to problem #1(c)

   (d) $f(t) = t$

   Answer: $\dfrac{1}{s^2}$ provided $s > 0$

   Click here to see video solution to problem #1(d)

   (e) $f(t) = t^2$

   Answer: $\dfrac{2}{s^3}$ provided $s > 0$

   Click here to see video solution to problem #1(e)

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2. Find the inverse Laplace transform of the following functions.

(a) $F(s) = \dfrac{4}{(s-2)^5}$

Answer: $\dfrac{e^{2t} \cdot t^4}{6}$

Click here to see video solution to problem #2(a)

(b) $F(s) = \dfrac{8s^2 - 4s + 12}{s(s^2+4)}$

Answer: $3 + 5\cos(2t) - 2\sin(2t)$

Click here to see video solution to problem #2(b)

(c) $F(s) = \dfrac{2s-3}{s^2+2s+10}$

Answer: $f(t) = 2e^{-t}\cos(3t) - \dfrac{5}{3}e^{-t}\sin(3t)$

Click here to see video solution to problem #2(c)

Video errata: In part (c), at the 20:29 mark, there is a sign error. The line should say $2 \cdot \dfrac{s+1}{(s+1)^2+3^2} - \dfrac{5}{3} \cdot \dfrac{3}{(s+1)^2+3^2}$. We then have that $f(t) = 2e^{-t}\cos(3t) - \dfrac{5}{3}e^{-t}\sin(3t)$.

3. Use the Laplace transform to solve the given initial value problem

(a) $y'' + 3y' + 2y = 4t, \qquad y(0) = 1, \quad y'(0) = 0.$

Answer: $-3 + 2t + 6e^{-t} - 2e^{-2t}$

Click here to see video solution to problem #3(a)

(b) $y'' + 9y = \cos 2t, \qquad y(0) = 0, \quad y'(0) = 1.$

Answer: $\dfrac{1}{5}\cos(2t) - \dfrac{1}{5}\cos(3t) + \dfrac{1}{3}\sin(3t)$

Click here to see video solution to problem #3(b)

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(c) $y'' - 2y' + 2y = e^{-t}, \qquad y(0) = 0, \quad y'(0) = 1.$

Answer: $y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{5}e^{-t} - \frac{1}{5}e^{t}\cos(t) + \frac{7}{5}e^{t}\sin(t)$

Click here to see video solution to problem #3(c)

Video errata: In part (c), at the 46:58 mark, we should have that $C = \frac{8}{5}$. We then have that $Y(s) = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+2} = \frac{1/5}{s+1} + \frac{1}{5}\cdot\frac{-s+8}{(s-1)^2+1}$. Moving forward, we have $Y(s) = \frac{1}{5}\cdot\frac{1}{s+1} - \frac{1}{5}\cdot\frac{s-1}{(s-1)^2+1} + \frac{7}{5}\cdot\frac{1}{(s-1)^2+1}$. Our final answer is then $y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{5}e^{-t} - \frac{1}{5}e^{t}\cos(t) + \frac{7}{5}e^{t}\sin(t)$. A note of this correction is made at the 57:39 mark.

4. Express $f(t)$ in terms of the unit step function $u_c(t)$ and find its Laplace transform.

(i) $f(t) = \begin{cases} t^2, & 0 \leqslant t < 2 \\ e^t, & 2 \leqslant t \end{cases}$

Answer: $\frac{2}{s^3} + e^{-2s}\left(\frac{e^2}{s-1} - \frac{2}{s^3} - \frac{4}{s^2} - \frac{4}{s}\right)$

Click here to see video solution to problem #4(i)

(ii) $f(t) = \begin{cases} 2, & 0 \leqslant t < 3 \\ 5t^2, & 3 \leqslant t < 8 \\ 3\cos(t-8), & 8 \leqslant t \end{cases}$

Answer: $\frac{2}{s} + e^{-3s}\left(\frac{10}{s^3} + \frac{30}{s^2} + \frac{43}{s}\right) + e^{-8s}\left(\frac{3s}{s^2+1} - \frac{10}{s^3} - \frac{80}{s^2} - \frac{320}{s}\right)$

Click here to see video solution to problem #4(ii)

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