



WEEK IN REVIEW SESSION #8 (SECTIONS 6.4-6.6)

1. Solve the following initial value problem using the Laplace transform:

$$y'' + 2y' + 5y = \sin(t) + u_\pi(t) \cos(t - \pi), \quad y(0) = 0, \quad y'(0) = 0.$$

2. Find the solution of the initial value problem

$$y'' - y = 4\delta(t - 2) + t^2, \quad y(0) = 0, \quad y'(0) = 2.$$

3. A 2-kg mass is attached to a spring with the Hooke's constant of 3 N/m, and it is subject to moving in a medium with a damping constant of 5 N-s/m. The mass is initially displaced +0.5 m from its equilibrium position and is released. Then after 2 seconds, a hammer hits the mass in such a way that its velocity suddenly changes by 5m/s in the positive direction (i.e., $\Delta v = +5$ m/s).

(a) Find the impulse of the hammer and its units.

(b) Write the hammer force using an appropriate Dirac delta function. Note that your force must result in the same impulse value you found in part (a).

(c) Set up an IVP and find the position $x(t)$ of the mass for $t \geq 0$.

4. Find the following convolutions using the definition only.

(a) $e^t * e^{3t}$

(b) $t * t^n$, where $n = 0, 1, 2, \dots$



5. Using the Laplace transform (instead of the definition), compute the following convolutions.
- (a) $u_a(t) * u_b(t)$

 - (b) $t^n * t^m$, $n, m = 0, 1, 2, \dots$
6. In each of the following cases find a function (or a generalized function) $g(t)$ that satisfies the equality for $t \geq 0$.
- (a) $t * g(t) = t^4$

 - (b) $1 * 1 * g(t) = t^2$

 - (c) $1 * g(t) = 1$
7. Write the inverse Laplace transform of the function $F(s) = \frac{s}{(s+1)^2(s+4)^3}$ in terms of a convolution integral.
8. Solve the initial value problem $y'' - 2y' - 3y = g(t)$, $y(0) = 1$, $y'(0) = -3$.