Week in Review Session ♯9 Exam 2 Review

This document contains the answers and video solutions to the posed problems. Click the red box to watch the video solution. You can also watch all videos by viewing the Session 9 playlist. In the event that this weekly review handout is updated, the Session video playlist will reflect the most updated problem set. Closed captions are available for all videos and the speed of the videos may be adjusted inside of "Settings" or the cog in the bottom right corner.

Review for Exam 2 (Please note that this review may not cover all sections of your Exam 2)

1. (3.3-3.5) Find the form of a particular solution for each of the following nonhomogeneous equations.

   (a) $y'' - 2y' = (x^2 - 3x + 1)e^{2x} + 3x\cos(2x) + x^2$

   Answer: $y_p = x(Ax^2+Bx+C)e^{2x} + (Dx+E)\cos(2x) + (Fx+G)\sin(2x) + x(Hx^2+Ix+J)$

   Click here to see video solution to problem #1(a)

   (b) $y'' + 6y' + 5y = e^{-3t} + t\sin(2t) + t^2 e^{-3t}\cos(2t)$

   Answer:
   $y_p = Ae^{-3t} + (Bt+C)\sin(2t) + (Dt+E)\cos(2t) + t\left((Ft^2 + Gt + H)\cos(2t) + (It^2 + Jt + K)\sin(2t)\right)$

   Click here to see video solution to problem #1(b)

2. (3.6) Find the general solution of the equation. $4y'' - 8y' + 5y = e^x \tan^2(x/2)$

   Answer: $y = e^x(c_1\cos(\frac{x}{2}) + c_2\sin(\frac{x}{2}) + \sin(\frac{x}{2})\ln|\sec(\frac{x}{2}) + \tan(\frac{x}{2})| - 2)$

   Click here to see video solution to problem #2

3. (6.1) Use the definition of the Laplace transform to find the Laplace transform of the following function:

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2 - t, & 1 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases}$$

   Answer: $\dfrac{1}{s^2} - \dfrac{2}{s^2}e^{-s} + \dfrac{1}{s^2}e^{-2s}$

   Click here to see video solution to problem #3

4. (6.3) Find the Laplace transform of the above function using Heaviside's unit step functions.

   Answer: The final answer is still $\frac{1}{s^2} - \frac{2}{s^2}e^{-s} + \frac{1}{s^2}e^{-2s}$.

   Click here to see video solution to problem #4

5. (6.2, 6.3) Find the inverse Laplace transform of the following functions

   (a) $F(s) = \dfrac{2s^2+1}{s^4+3s^3-4s^2}$.

   Answer: $-\frac{3}{16} - \frac{1}{4}t - \frac{33}{80}e^{-4t} + \frac{3}{5}e^t$

   Click here to see video solution to problem #5(a)

   (b) Find the inverse Laplace transform of the function $F(s) = \dfrac{s+3se^{-5s}}{s^2-4s+3}$

   Answer: $-\frac{1}{2}e^t + \frac{3}{2}e^{3t} + u_5(t)\left(-\frac{3}{2}e^{t-5} + \frac{9}{2}e^{3(t-5)}\right)$

   Click here to see video solution to problem #5(b)

6. (6.3, 6.4) Solve the following initial value problem using the Laplace transform:

$$y'' + y' + \frac{5}{4}y = \begin{cases} \sin t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases} \quad ; \quad y(0) = 0, \quad y'(0) = 0$$

   Answer:
   $y(t) = h(t) + u_\pi(t)h(t-\pi)$, where $h(t) = -\frac{16}{17}\cos(t) + \frac{4}{17}\sin(t) + \frac{1}{17}e^{-\frac{1}{2}t}[16\cos(t) + 4\sin(t)]$

   Click here to see video solution to problem #6

7. (6.5) Use the definition of the Laplace transform to find the Laplace transform of the following function

$$f(t) = \delta(t-2)(4t^2 - \cos(\pi t))$$

   Answer: $F(s) = 15e^{-2s}$

   Click here to see video solution to problem #7

8. (6.6) Find the Laplace transform of

(a) $f(t) = \int_0^t (t-\tau)e^{3\tau}\, d\tau$

Answer: $\dfrac{1}{s^2(s-3)}$

Click here to see video solution to problem #8(a)

(b) $f(t) = \int_0^t \left(e^{\tau}\sin(t-\tau)\right)\, d\tau$

Answer: $\dfrac{1}{(s-1)(s^2+1)}$

Click here to see video solution to problem #8(b)