Note ♯10: Sections 6.5-6.6

Problem 1. Find the values.

(a)

$$\int_{-\infty}^{\infty} \delta(t-3)dt$$

(b)

$$\int_{-\infty}^{\infty} \delta(t-3)t^2\cos(t-2)dt$$

(c)

$$\mathcal{L}\{\delta(t-3)(3t+1)\}$$

Problem 2. Find the solution of the given initial value problem.

(a)

$$y'' + y = \delta(t - 2\pi)\cos t; \quad y(0) = 0, \quad y'(0) = 1$$

(b)

$$y'' + 2y' + 2y = \delta(t - \pi); \quad y(0) = 1, \quad y'(0) = 0$$

Problem 3. (a) Find the Laplace transform of the given function.

$$f(t) = \int_0^t (t-\tau)^2 \cos(2\tau) d\tau$$

(b) Find the inverse Laplace transform of the given function by using the convolution theorem.

$$F(s) = \frac{s}{(s+1)(s^2+4)}$$

Problem 4. Express the solution of the given initial value problem in terms of a convolution integral.

$$y'' + 3y' + 2y = \cos(\alpha t); \quad y(0) = 1, \quad y'(0) = 0$$

Note ♯10: Second Midterm Review

Problem 5. Find the solutions of the differential equations.

(a) $y'' - y' - 2y = 0$

(b) $y'' + 6y' + 9y = 0$

(c) $y'' - 2y' + 5y = 0$

Problem 6. Find the solution of the given initial value problem using the method of undetermined coefficients.

$$y'' + y' - 2y = t^2 + 3e^t, \quad y(0) = 0, y'(0) = 2$$

Problem 7. Find the general solution of the given differential equation using the variation of parameters.

$$y'' - 5y' + 6y = e^{-2t}$$

Problem 8. A spring-mass system has a spring constant of 3N/m. A mass of 2kg is attached to the spring, and the motion takes place in a viscous fluid that offers a resistance numerically equal to the magnitude of the instantaneous velocity. If the system is driven by an external force of $(3\cos(3t) - 2\sin(3t))$N, determine the steady-state response. Express your answer in the form $R\cos(\omega t - \delta)$.

Problem 9. Find the Laplace transform using the definition of the Laplace transform:

$$f(t) = \begin{cases} 1, & 0 \leq t < 5, \\ t, & 5 \leq t \end{cases}$$

Problem 10. Express $f(t)$ in terms of the unit step function $u_c(t)$ and find the Laplace transform.

$$f(t) = \begin{cases} t, & 0 \leq t < 2, \\ 7 - t, & 2 \leq t < 5, \\ t^2, & 5 \leq t \end{cases}$$

Problem 11. Find the solution of the given initial value problem.

$$y'' + y' + \frac{5}{4}y = f(t); \quad y(0) = 0, y'(0) = 0,$$

$$f(t) = \begin{cases} t, & 0 \le t < \pi/2 \\ \pi/2, & \pi/2 \le t < \infty. \end{cases}$$