Note ♯13: Sections 7.4-7.6, 7.8

Problem 1. $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}\mathbf{x}; \quad \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t}, \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

a. Show that the given functions are solutions of the given system of differential equations.
b. Show that $\mathbf{x} = c_1\mathbf{x} + c_2\mathbf{x}$ is also a solution of the given system for any values of c_1 and c_2.
c. Show that the given functions form a fundamental set of solutions of the given system.
d. Find the solution of the given system that satisfies the initial condition $\mathbf{x}(0) = (1, 2)^T$.
e. Find $W\left[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}\right](t)$.

Problem 2.

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x}$$

a. Draw a direction field.

b. Find the general solution of the given system of equations and describe the behavior of the solution as $t \to \infty$.

c. Plot a few trajectories of the system.

Problem 3.

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$$

a. Draw a direction field.

b. Find the general solution of the given system of equations and describe the behavior of the solution as $t \to \infty$.

c. Plot a few trajectories of the system.

Problem 4.

$$\mathbf{x}' = \begin{pmatrix} -\frac{3}{2} & 1 \\ -\frac{1}{4} & -\frac{1}{2} \end{pmatrix} \mathbf{x}$$

a. Draw a direction field.

b. Find the general solution of the given system of equations and describe the behavior of the solution as $t \to \infty$.

c. Plot a few trajectories of the system.