Note #: Sections 2.4-2.5

Problem 1. Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

a. 
\[(4 - t^2)y' + 2ty = 3t^2, \quad y(-3) = 1\]

b. 
\[(t - 3)y' + (\ln t)y = 2t, \quad y(1) = 2\]
Problem 2. State where in the ty-plane the hypotheses of Theorem 2.4.2 (existence and uniqueness theorem for nonlinear equations) are satisfied.

a. 
\[ y' = (t^2 + y^2)^{3/2} \]

b. 
\[ y' = \frac{1 + t^2}{3y - y^2} \]
Problem 3. Solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value $y_0$.

a. 
\[ y' + y^3 = 0, \quad y(0) = y_0 \]

b. 
\[ y' = \frac{t^2}{y(1 + t^3)}, \quad y(0) = y_0 \]
Problem 4.

\[ \frac{dy}{dt} = (y - 4)(y - 2)(y + 1) \]

a. Determine the critical (equilibrium) points.
b. Sketch the graph of \( f(y) \) versus \( y \).
c. Draw the phase line.
d. Classify equilibrium points.
e. Sketch several graphs of solutions in the ty-plane.
Problem 5.

\[ \frac{dy}{dt} = (y - 3)^2(y - 1)(y + 2)^2 \]

a. Determine the critical (equilibrium) points.
b. Draw the phase line.
c. Classify equilibrium points.
d. Sketch several graphs of solutions in the \(ty\)-plane.
Problem 6. Another equation that has been used to model population growth is the Gompertz equation

\[ \frac{dy}{dt} = ry \ln \frac{K}{y}, \]

where \( r \) and \( K \) are positive constants.

a. Sketch the graph of \( f(y) \) versus \( y \), find the critical points, and determine whether each is asymptotically stable or unstable.

b. For \( 0 \leq y \leq K \), determine where the graph of \( y \) versus \( t \) is concave up and where it is concave down.

c. Solve the Gompertz equation subject to the initial condition \( y(0) = y_0 \). Hint: You may wish to let \( u = \ln(y/K) \).