



NOTE #5: SECTIONS 3.3-3.4

Problem 1. Find the solutions of the given differential equations.

a. $y'' - 2y' + 6y = 0$

b. $y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, y'(\pi/2) = 2$

Problem 2. Use the method of reduction of order to find a second solution of the given differential equation.

$$t^2y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}$$

Problem 3. Solve the initial value problem.

$$y'' - 6y' + 9y = 0, \quad y(0) = 0, y'(0) = 2$$

NOTE #5: MIDTERM REVIEW

Problem 4. Solve the initial value problem

$$xy' + (2x - 3)y = 4x^4, \quad y(1) = 0.$$

Problem 5. Find the general solution in explicit form.

$$\frac{1}{x} \frac{dy}{dx} = y^2 \sin x.$$

Problem 6. What is the maximum interval on which the solution of the initial value problem exists and is unique by the existence/uniqueness theorem?

$$(t^2 - 1)y' + t^2y = t - 3, \quad y(-2) = 2021$$

Problem 7.

$$dy/dt = (y - 2)^2(y + 1)(y + 3)$$

- a. Determine the critical (equilibrium) points, and classify each one as asymptotically stable, unstable, or semistable.
- b. Draw some sample solution curves.

Problem 8. A tank contains 200 L of saline water with the salt concentration 3 g/L. A mixture containing a concentration of 5 g/L of salt enters the tank at a rate of 10 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression for the amount of salt in the tank at time t .

Problem 9. Determine whether the equation is exact. If it is exact, find the general solution in explicit form.

$$(2xy - \sin x) + (x^2 + 2y)y' = 0$$

Problem 10. Find the general solution of the differential equation.

$$y'' - 7y' + 12y = 0$$

Problem 11. Show that the functions y_1 and y_2 form a fundamental set of solutions and find the general solution.

$$t^2 y'' - 2y = 0, \quad t > 0$$

$$y_1(t) = t^2, \quad y_2(t) = t^{-1}$$