Problem 1. A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10 \sin(t/2)$ N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s,

a. formulate the initial value problem describing the motion of the mass.
b. Find the solution of the initial value problem.
c. Identify the transient and steady-state parts of the solution.
Problem 2. Consider the vibrating system described by the initial value problem

\[ u'' + u = 3\cos(\omega t), \quad u(0) = 1, \quad u'(0) = 1. \]

a. Find the solution for \( \omega \neq 1 \).
b. Find the solution for $\omega = 1$. 
Problem 3. Find the Laplace transform of the function.
   a. $f(t) = t^2$
b. \( f(t) = \cos(at) \), where \( a \) is a real constant.
c. \( f(t) = \begin{cases} 
  t, & 0 \leq t < 1 \\
  2 - t, & 1 \leq t < 2 \\
  0, & 2 \leq t < \infty 
\end{cases} \)
Problem 4. Find the inverse Laplace transforms

(a) \( F(s) = \frac{3}{s^2+4} \)

(b) \( F(s) = \frac{1-2s}{s^2+4s+5} \)
(c) \( F(s) = \frac{2s-3}{s^2-4} \)

(d) \( F(s) = \frac{8s^2-4s+12}{s(s^2+3)} \)