



NOTE #8: SECTION 3.8 - 6.1

Problem 1. A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10 \sin(t/2)$ N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s,

- a. formulate the initial value problem describing the motion of the mass.
- b. Find the solution of the initial value problem.
- c. Identify the transient and steady-state parts of the solution.

Problem 2. Consider the vibrating system described by the initial value problem

$$u'' + u = 3 \cos(\omega t), \quad u(0) = 1, u'(0) = 1.$$

- a. Find the solution for $\omega \neq 1$.

b. Find the solution for $\omega = 1$.

4

Problem 3. Find the Laplace transform of the function.

a. $f(t) = t^2$

b. $f(t) = \cos(at)$, where a is a real constant.

6

$$\text{c. } f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 2 - t, & 1 \leq t < 2 \\ 0, & 2 \leq t < \infty \end{cases}$$

Problem 4. Find the inverse Laplace transforms

(a) $F(s) = \frac{3}{s^2+4}$

(b) $F(s) = \frac{1-2s}{s^2+4s+5}$

8

$$(c) F(s) = \frac{2s-3}{s^2-4}$$

$$(d) F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$$