# NOTE ♯8: SECTION 6.2 - 6.4

**Problem 1.** Find the inverse Laplace transforms

(a) $F(s) = \frac{3}{s^2+4}$

(b) $F(s) = \frac{1-2s}{s^2+4s+5}$

(c) $F(s) = \frac{2s-3}{s^2-4}$

(d) $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$

**Problem 2.** Use the Laplace transform to solve the given initial value problem.

(a) $y'' + 3y' + 2y = e^{-t}; \quad y(0) = 0, \quad y'(0) = 1$

(b) $y^{(4)} - y = 0; \quad y(0) = 1, y'(0) = 0, \quad y''(0) = 1$

**Problem 3.** Express $f(t)$ in terms of the unit step function $u_c(t)$ and find the Laplace transform.

(a) $f(t) = \begin{cases} 0, & t < 2 \\ t - 2, & 2 \leq t < 4 \\ t^2 - 7t + 14, & t \geq 4 \end{cases}$

(b) $f(t) = \begin{cases} 0, & t < 2 \\ t - 4, & 2 \leq t < 4 \\ t^2 - 9t + 5, & t \geq 4 \end{cases}$

**Problem 4.** Find the inverse Laplace transform of the given function.

(a)

$$F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s}}{s}$$

(b)

$$F(s) = \frac{2}{s^2 - 4s + 13}$$

(c)

$$F(s) = \frac{2(s-1)e^{-2s}}{s^2 - 4s + 13}$$

**Problem 5.** Find the solution of the given initial value problem.

(a)

$$y'' + 2y' + 2y = h(t); \quad y(0) = 0, \quad y'(0) = 1$$

$$h(t) = \begin{cases} 1, & \pi \le t < 2\pi \\ 0, & 0 \le t < \pi \text{ or } t \ge 2\pi \end{cases}$$

(b)

$$y'' + 4y = \sin t - u_{2\pi}(t)\sin(t - 2\pi); \quad y(0) = 0, \quad y'(0) = 0$$