Note # 9: Statistical Inference for Numerical Variables

Problem 1. A researcher is interested in knowing the average amount of time individuals in Cincinnati, Ohio spend commuting. He takes a random sample of 100 Cincinnati residents and asks them what their typical commute time is. He finds the average length of time for the residents in his sample is 32 minutes, with a standard deviation of 12 minutes. Create a 98% confidence interval for the average commute time of all Cincinnati residents.

- **a.** Construct the 98% confidence interval.
- **b.** Interpret your confidence interval from part a.

Answer:

$\bar{x} = 32, \quad s = 12, \quad n = 100, \quad t^* = 2.374, \quad df = n - 1 = 100 - 1 = 99 \ (use\ df = 80)$

a. $CI: \ \bar{x} \pm t^* \frac{s}{\sqrt{n}} = 32 \pm (2.374)\frac{12}{\sqrt{100}} = 32 \pm (2.374)(1.2)$

$= 32 \pm 2.8488 \ \rightarrow \ 98\% \ CI: (\mathbf{29.1512}, \mathbf{34.8488})$

b. We are 98% confident that the true average commute time for all residents of Cincinnati, Ohio is between 29.1512 minutes and 24.8488 minutes.

Problem 2. A researcher is interested in knowing the average amount of time individuals in Cincinnati, Ohio spend commuting. He takes a random sample of 100 Cincinnati residents and asks them what their typical commute time is. He finds the average length of time for the residents in his sample is 32 minutes, with a standard deviation of 12 minutes. The researcher believes the average commute time of all Cincinnati residents is greater than 30 minutes. Conduct a hypothesis test at the 0.02 significance level to test this.

- **a.** What are the hypotheses?
- **b.** What is the significance level?
- **c.** What is the value of the test statistic?
- **d.** What is the p-value?
- **e.** What is the correct decision?
- **f.** What is the appropriate conclusion/interpretation?
- **g.** Does the hypothesis test agree with the confidence interval from question 1?

Answer:

- **a.** $H_0: \mu = 30$ vs. $H_A: \mu > 30$
- **b.** $\alpha = 0.02$
- **c.** $TS = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{32-30}{12/\sqrt{100}} = \frac{2}{1.2} = \mathbf{1.6667}$
- **d.** $TS = 1.6667\,,\ df = n-1 = 100-1 = 99\ (use\ df = 80) \rightarrow\ 0.025 < p-value < 0.05$
- **e.** $\because 0.025 < p-value < 0.05 \ \rightarrow\ p-values > 0.025 >\ \alpha = 0.02,\ \therefore$ **Fail to reject H_0.**
- **f.** The data does not provide statistically significant evidence that the average commute time for all Cincinnati residents is greater than 30 minutes.
- **g.** 98% CI: $(\mathbf{29.1512}\,, \mathbf{34.8488})$. Yes, null value (30) is in the interval and we failed to reject H_0.

Problem 3. The Human Toxome Project (HTP) is working to understand the scope of industrial pollution in the human body. Industrial chemicals may enter the body through pollution or as ingredients in consumer products. In October 2008, scientists at HTP tested cord blood samples for 20 newborn infants in the United States. The cord blood of the "In utero/newborn" group was tested for 430 industrial compounds, pollutants, and other chemicals, including chemicals linked to brain and nervous system toxicity, immune system toxicity, reproductive toxicity, and fertility problems. The researchers recorded the number of these targeted chemicals that were found in each infant's cord blood. The average number of targeted chemicals found was 127.45, with a standard deviation of 25.965. Create a 90% confidence interval for the average number of targeted chemicals in all infants' cord blood.

- **a.** Construct the 90% confidence interval.
- **b.** Interpret your confidence interval from part a.

Answer:

$\bar{x} = 127.45, \quad s = 25.965, \quad n = 20, \quad df = n - 1 = 20 - 1 = 19, \quad t^* = 1.729$

a. $CI\colon \ \bar{x} \pm t^* \frac{s}{\sqrt{n}} = 127.45 \pm (1.729)\frac{25.965}{\sqrt{20}} = 127.45 \pm (1.729)(5.806)$

$= 127.45 \pm 10.038 \quad \rightarrow \quad 90\%\ CI\colon (\mathbf{117.412}\, , \mathbf{137.488})$

b. We are 90% confident that the true average number of targeted chemicals found in infants' cord blood (US, 2008) is between 117.412 and 137.488.

Problem 4. The Human Toxome Project (HTP) is working to understand the scope of industrial pollution in the human body. Industrial chemicals may enter the body through pollution or as ingredients in consumer products. In October 2008, scientists at HTP tested cord blood samples for 20 newborn infants in the United States. The cord blood of the "In utero/newborn" group was tested for 430 industrial compounds, pollutants, and other chemicals, including chemicals linked to brain and nervous system toxicity, immune system toxicity, reproductive toxicity, and fertility problems. The researchers recorded the number of these targeted chemicals that were found in each infant's cord blood. The average number of targeted chemicals found was 127.45, with a standard deviation of 25.965. In 2000, the average number of targeted chemicals found in infant's cord blood was 120. Conduct a hypothesis test at the 0.10 significance level to test if the average number has changed.?

- **a.** What are the hypotheses?
- **b.** What is the significance level?
- **c.** What is the value of the test statistic?
- **d.** What is the p-value?
- **e.** What is the correct decision?
- **f.** What is the appropriate conclusion/interpretation?
- **g.** Does the hypothesis test agree with the confidence interval from question 3?
- **h.** How would the hypothesis test have changed if we wanted to see if the number of targeted chemicals had increased, instead of seeing if it had changed?

Answer:

- **a.** $H_0: \mu = 120$ vs. $H_A: \mu \neq 120$
- **b.** $\alpha = 0.10$
- **c.** $TS = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{127.45-120}{25.965/\sqrt{20}} = \frac{7.45}{5.806} = \mathbf{1.283}$
- **d.** $TS = 1.283\,,\quad df = n-1 = 20-1 = 19 \rightarrow\; 0.20 < p-value < 0.30$
- **e.** $\because\; 0.20 < p-value < 0.30 \;\rightarrow\; p-values > 0.20 > \alpha = 0.10,\; \therefore$ **Fail to reject H_0.**
- **f.** The data does not provide statistically significant evidence that true average number of targeted chemicals found in infants' cord blood (US, 2008) is different from 120 (2000 baseline).
- **g.** 90% CI: $(\mathbf{117.412}\,,\mathbf{137.488})$. Yes, null value (120) is in the interval. Failed to reject H_0.
- **h.** One sided test → alternative hypothesis would look different → $p-value$ would have changed $0.10 < p-value < 0.15 \rightarrow p-values > 0.10 > \alpha = 0.10$, so decision and conclusion stay the same.

Problem 5. Suppose we want to determine if there is a difference between the price of a textbook on Amazon and the price of the same textbook at the UCLA bookstore. We sampled 201 UCLA courses. Of those, 68 required books could be found on Amazon. For each of these 68 books, we recorded the price of the book at the UCLA bookstore, the price of the book on Amazon, and the difference between the two (UCLA bookstore minus Amazon). The average difference in our sample was $3.58, with a standard deviation of $13.42. Create a 96% confidence interval for the average difference between the UCLA bookstore price and the Amazon price.

- **a.** Construct the 96% confidence interval.
- **b.** Interpret your confidence interval from part a.

Answer:

$\bar{x}_{diff} = 3.58, \quad s_{diff} = 13.42, \; n_{diff} = 68, \qquad df = n_{diff} - 1 = 68 - 1 = 67 \; (use\; df = 60), \qquad t^* = 2.099$

a. $CI: \;\; \bar{x}_{diff} \pm t^* \frac{s_{diff}}{\sqrt{n_{diff}}} = 3.58 \pm (2.099)\frac{13.42}{\sqrt{68}} = 3.58 \pm (2.099)(1.6274)$

$= 3.58 \pm 3.4159 \;\; \rightarrow \;\; 96\%\; CI: (\$\mathbf{0.16}\,, \$\mathbf{7.00})$

b. We are 96% confident that the true average difference between the Amazon textbook price and the UCLA bookstore textbook price is between $^{\$}0.16$ and $^{\$}7.00$.

Problem 6. Suppose we want to determine if there is a difference between the price of a textbook on Amazon and the price of the same textbook at the UCLA bookstore. We sampled 201 UCLA courses. Of those, 68 required books could be found on Amazon. For each of these 68 books, we recorded the price of the book at the UCLA bookstore, the price of the book on Amazon, and the difference between the two (UCLA bookstore minus Amazon). The average difference in our sample was $3.58, with a standard deviation of $13.42. Conduct a hypothesis test at the 0.04 significance level to test this.

- **a.** What are the hypotheses?
- **b.** What is the significance level?
- **c.** What is the value of the test statistic?
- **d.** What is the p-value?
- **e.** What is the correct decision?
- **f.** What is the appropriate conclusion/interpretation?
- **g.** Does the hypothesis test agree with the confidence interval from question 5?

Answer:

- **a.** $H_0: \mu_{diff} = 0$ vs. $H_A: \mu_{diff} \neq 0$
- **b.** $\alpha = 0.04$
- **c.** $TS = \frac{\bar{x}_{diff} - \mu_{diff}}{s_{diff}/\sqrt{n_{diff}}} = \frac{3.58-0}{13.42/\sqrt{68}} = \frac{3.58}{1.6274} = \mathbf{2.1998}$
- **d.** $TS = 2.1998\,,\ \ df = n - 1 = 68 - 1 = 67\ (use\ df = 60)\ \rightarrow\ 0.02 < p - value < 0.04$
- **e.** $\because 0.02 < p - value < 0.04\ \rightarrow\ p - values < 0.04 < \alpha = 0.04,\ \therefore$ **Reject H_0.**
- **f.** The data does provide statistically significant evidence that, on average, there is a difference between the price of a textbook on Amazon, and the price of the same textbook at the UCLA bookstore. Based on our sample, we believe books tend to be more expensive at the UCLA bookstore (on average)
- **g.** $96\%\ CI: (\$\mathbf{0.16}\,, \$\,\mathbf{7.00})$. Yes, null value (0) is not in the interval and we rejected H_0.

Problem 7. A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Eight randomly selected individuals were asked to rank their pain both before and after being hypnotized. A lower score indicates less pain. The average difference (after-before) for the sample was -3.13, with a standard deviation of 2.91. Create a 90% confidence interval for the average difference in pain levels.

a. Construct the 90% confidence interval.

b. Interpret your confidence interval from part a.

Answer:

$\bar{x}_{diff} = -3.13, \quad s_{diff} = 2.91, \; n_{diff} = 8, \qquad df = n_{diff} - 1 = 8 - 1 = 77\,, \qquad t^* = 1.895$

a. $CI\!: \;\; \bar{x}_{diff} \pm t^* \frac{s_{diff}}{\sqrt{n_{diff}}} = -3.13 \pm (1.895)\frac{2.91}{\sqrt{8}} = -3.13 \pm (1.895)(1.029)$

$= -3.13 \pm 1.95 \quad \rightarrow \quad 90\%\; CI\!: (\mathbf{-5.08}\,, \mathbf{-1.18})$

b. We are 90% confident that the true average difference in pain scores after-before hypnotism is between -5.08 and -1.18.

Problem 8. A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Eight randomly selected individuals were asked to rank their pain both before and after being hypnotized. A lower score indicates less pain. The average difference (after-before) for the sample was -3.13, with a standard deviation of 2.91. Conduct a hypothesis test at the 0.04 significance level to test if hypnotism changes the pain level.

a. What are the hypotheses?
b. What is the significance level?
c. What is the value of the test statistic?
d. What is the p-value?
e. What is the correct decision?
f. What is the appropriate conclusion/interpretation?
g. Does the hypothesis test agree with the confidence interval from question 7?
h. How would the hypothesis test have changed if we wanted to determine if hypnotism reduced pain (instead of if it changes pain levels)?

Answer:

a. $H_0: \mu_{diff} = 0$ vs. $H_A: \mu_{diff} \neq 0$

b. $\alpha = 0.10$

c. $TS = \frac{\bar{x}_{diff} - \mu_{diff}}{s_{diff}/\sqrt{n_{diff}}} = \frac{-3.13-0}{2.91/\sqrt{8}} = \frac{-3.13}{1.029} = \mathbf{-3.042}$

d. $TS = -3.042\,,\ \ df = n - 1 = 8 - 1 = 7 \ \rightarrow\ 0.01 < p - value < 0.02$

e. $\because 0.01 < p - value < 0.02 \ \rightarrow\ p - values < 0.02 < \alpha = 0.10,\ \therefore$ **Reject H_0**.

f. The data provide statistically significant evidence that there is a true average difference between the pain scores before and after hypnotism. Based on our sample, we believe the scores decrease after hypnotism (on average).

g. $90\%\ CI$: (: $(\mathbf{-5.08}\,, \mathbf{-1.18})$. Yes, our null value (0) is not in the interval and we rejected H_0.

h. One sided test $\rightarrow$ alternative hypothesis would change $\rightarrow p - value$ changes

$0.005 < p - value < 0.01$, so decision and conclusion stay the same.