MATH 140: Week-In-Review 3
(2.4, Review Questions over Ch 1 & Ch 2)

Problem 1 *Write the augmented matrix corresponding to the given system of linear equations.*

$7x - 8y + z = 90$
$5y - 10z = 87 + 3x$
$2z = 6x - 50$
$-9y + 54 = 7x$

Problem 2 *What system of linear equations would the following augmented matrix represent? (Assume variables x and y, respectively.)*

$$\left[\begin{array}{cc|c} 2 & 15 & 50 \\ -3 & 7 & 20 \end{array}\right]$$

Problem 3 *Perform the given row operations in the Gauss-Jordan Elimination Method, and show the resulting matrices.*

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 10 \\ 0 & 4 & -8 & 12 \\ 0 & 2 & 3 & -8 \end{array}\right] \xrightarrow{1/4R_2 \longrightarrow R_2} \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$$

$$\xrightarrow{-2R_2+R_3 \longrightarrow R_3} \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$$

$$\xrightarrow{1/7R_3 \longrightarrow R_3} \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$$

Problem 4 *Pivot the following system about the element in row 3, column 3.*

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -5 & -4 \\ 0 & 0 & 3 & 9 \end{array}\right]$$

Problem 5 *Are the following augmented matrices in reduced row-echelon form?*

If YES, write the resulting simplified system and corresponding solution.
(Assume variables are x and y or x, y and z.)

If NO, write the next best row operation you would use in the Gauss-Jordan Elimination Method.

(a) $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & 6 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 7 \end{array}\right]$

(d) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$

(e) $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right]$

(f) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Problem 6 *Solve the following systems of linear equations. If there are infinitely many solutions, find both the parametric solution and one specific solution. State whether the system is independent, inconsistent, or dependent.*

(a) $2x + y = 8 - z$
$3y = x + z + 10$
$5y - 5z = x + 18$
$y - 5z = 2x$

(b) $4x - 2y + z = 30$
$-x + 5y - 2z = -10$
$2x + 8y - 3z = 15$

(c) $x + 2y + z = 10$
$-x + 4z = 20 + 3y$
$x + y = 40 - 6z$

(d) $-8x + 3y = -30$
$20x - 15y = 70$
$x - y = 2$

(e) $-2x + 5y - 3z = 30$
$x - y - 2z = 50$
$x - 7y + 12z = -210$

(f) $x + 2y + 3z - 4w = 10$
$2x - 4y + 2z + 6w = 20$
$3x - 10y + z + 16w = 30$

For the next three problems, set up a system of linear equations representing the given problem, and then find the solution to the problem.

Problem 7 *A company makes two types of toys: zappers and wompers. Each zapper takes 5 oz. of plastic and 4 minutes to manufacture. Each womper takes 3 oz. of plastic and 2 minutes to manufacture. There is available 5250 oz. of plastic and 60 hours of manufacturing time. If the company makes a total of 1650 toys and all resources are to be used, determine how many of each toy the company can make.*

Problem 8 *A company makes three different wallpapers, each containing two dyes: green and blue. Each roll of Wallpaper I uses 2 units of green and 1 unit of blue dye. Each roll of Wallpaper II uses 4 units of green and 1 unit of blue dye. Each roll of Wallpaper III uses 1 unit of each color of dye. There are 600 total units of green and 300 total units of blue dye available. If all the dye is used and there are the same number of rolls of Wallpaper I made as the sum of the number of rolls of the other types of wallpaper combined, how much of each type of wallpaper is made?*

Problem 9 *A fast food restaurant has three sizes of drinks: small, medium, and large. The price of each is \$0.79, \$0.99, and \$1.29, respectively. On a tax-free holiday, a total of 80 drinks are sold, bringing in a revenue of \$92.20. On that day, twice as many medium drinks were sold as small drinks. How many of each type of drink were sold?*

For the following problem, set up a system of equations representing the following problem and then find the solution to the problem. If the solution is parametric, then tell what restrictions can be placed on the parameter(s).

Problem 10 *A moving company wants to purchase a fleet of 24 trucks with a combined carrying capacity of 250,000 pounds. Trucks with three different carrying capacities are available: 6,000 pounds, 8,000 pounds, and 18,000 pounds. How many of each type of truck should be purchased?*

Problem 11 *Solve the following matrix equation for the variables u, x and y. If this is not possible, then explain why not.*

$$\begin{bmatrix} 5 \\ 0 \end{bmatrix} \left(\begin{bmatrix} 3 \\ 4+y \end{bmatrix}^T + 3 \begin{bmatrix} 1 & -8 \end{bmatrix} \right) = \begin{bmatrix} (x+7) & 3 \\ (u+x) & 0 \end{bmatrix}$$

Problem 12 *A new movie opened and adult ticket sales from three theaters in July and August are given by matrices J and A, below.*

$$J = \begin{array}{r} \\ Theater C \\ Theater H \\ Theater S \end{array} \begin{array}{c} Sales \\ \left[\begin{array}{c} 7500 \\ 4000 \\ 1000 \end{array}\right] \end{array} \qquad A = \begin{array}{r} \\ Theater C \\ Theater H \\ Theater S \end{array} \begin{array}{c} Sales \\ \left[\begin{array}{c} 5000 \\ 3500 \\ 2000 \end{array}\right] \end{array}$$

If adult tickets at Theaters C, H, and S are sold for \$8.00, \$7.25, and \$7.50, respectively, write a matrix P which can be used with J and A to determine the total revenue brought in from all three theaters over the months of July and August. Use the matrices to determine these revenues.

Problem 13

(a) *Find the equation of the line that passes through the* y*-intercept of the line* $4x + 2y = 7$ *and also passes through the* x*-intercept of the line* $y = 9x + 8$*.*

(b) *For the line found in part (a), for every decrease in* x *by 2 units, what is the corresponding change in* y*?*

Problem 14 *A sailor buys a yacht and after 1 year it is worth \$980,000, but after 10 years it is worth \$800,000.*

(a) Assuming the yacht is depreciating linearly, find the value (in dollars) of the yacht, V, as a function of the number of years the sailor has owned it, t.

(b) Assuming a scrap value of \$0, how long will the yacht have value?

Problem 15 *A company making blankets incurs a total cost of $3375 when producing 250 blankets and a total cost of $3750 when producing 300 blankets. The company sells the blankets for $20 each.*

(a) *Assuming a linear cost function, what are the fixed costs for this company?*

(b) *Find the company's linear profit function, $P(x)$, where x represents the number of blankets made and sold and P is measured in dollars.*

(c) *What is the break-even point for this company?*

Problem 16 *Given a supply function of* $3x - 11p + 45 = 0$ *where* x *represents the number of items made/sold (in units of a thousand) and* p *represents the price of the item (in dollars), and the fact that consumers will not buy the item being sold at a price above \$8, but will buy 14000 items at a price of \$4, what are the price and quantity which would satisfy both consumers and producers at the same time?*