Session 3: Review for Exam 1

(1) The daily cost, $C(x)$, (in dollars) a company incurs for making and selling x travel mugs is given the table below.

(a) Find the average rate of change of cost on the following intervals: $[97, 100]$, $[98, 100]$, $[99, 100]$. Round your answer to four decimals if necessary.

x	96	97	98	99	100	101	102	103	104
$C(x)$	117.98	118.49	118.99	119.5	120	120.5	121	121.49	121.98

(b) Use your answer from part (a) and the average rates of change on the intervals given below, to approximate the instantaneous rate of change of cost when 100 travel mugs are made and sold. Round your answer to four decimals if necessary.

- average rate of change on $[100, 104]$ is $\dfrac{121.98 - 120}{104 - 100} = \dfrac{1.98}{4} = 0.4950$ dollars per travel mug
- average rate of change on $[100, 103]$ is $\dfrac{121.49 - 120}{103 - 100} = \dfrac{1.49}{3} = 0.4967$ dollars per travel mug
- average rate of change on $[100, 102]$ is $\dfrac{121 - 120}{102 - 100} = \dfrac{1}{2} = 0.5$ dollars per travel mug
- average rate of change on $[100, 101]$ is $\dfrac{120.5 - 120}{101 - 100} = \dfrac{0.5}{1} = 0.5$ dollars per travel mug

(2) A company's daily profit when x gift baskets are made and sold is $P(x)$ dollars. Given $\dfrac{P(35 + h) - P(35)}{h} = 10 - h$,

(a) find and interpret the average rate of change of profit when the number of gift baskets made and sold is between 35 and 50.

(b) find and interpret the instantaneous rate of change of profit when 35 gift baskets are made and sold.

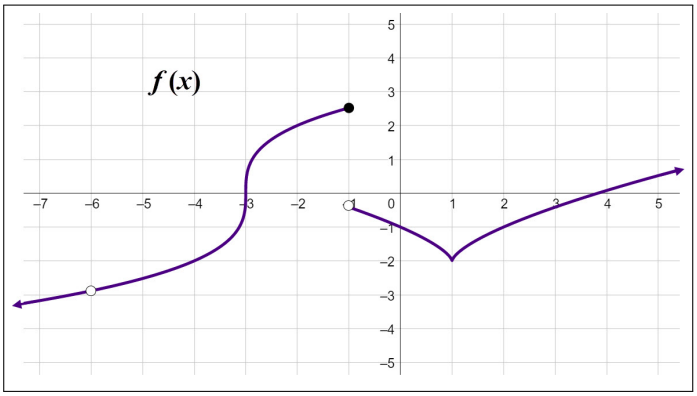
(3) Use algebraic methods to find the equation of the line tangent to $g(x) = \dfrac{2x}{3x + 5}$ at $a = -1$

(4) Find the $x-$value where the graph of the function $h(x) = 10\sqrt{9 - x}$ has a slope of -5.

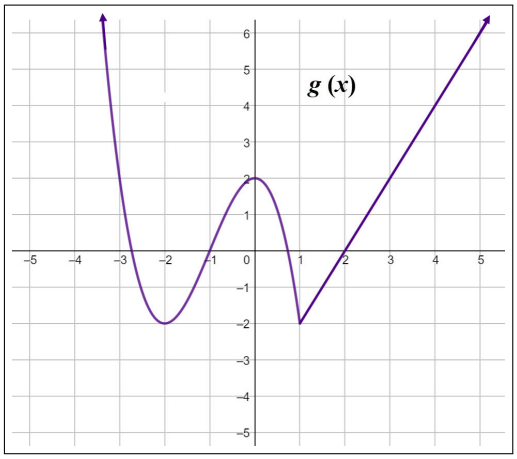
(5) Draw a graph of a function that has horizontal tangents at $x = -3,\ 2$, and 5.

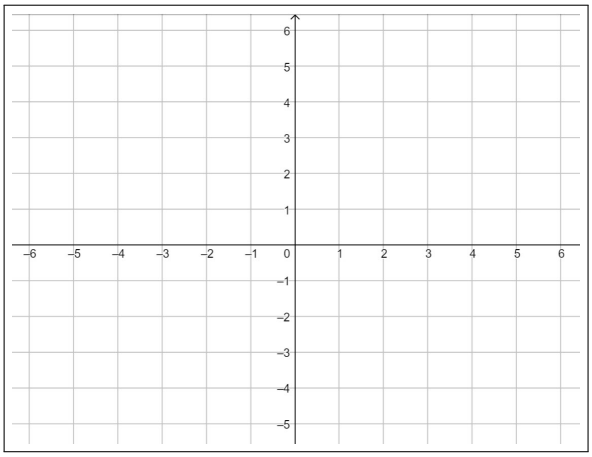
(6) Use the limiting definition of the derivative to find $f\,'(x)$ given $f(x) = 3x^2 - x$. Use your answer to determine where $f(x)$ has horizontal tangent lines.

(7) Use the graph of $f(x)$ below to find (a) values of c where $\lim_{x\to c} f(x)$ DNE, (b) the intervals on which $f(x)$ is continuous, and (c) the values of x where $f(x)$ is not differentiable.



(8) Given $g(x)$ below, use the blank grid to sketch a possible graph of $g\,'(x)$. (Note: There are multiple correct answers.)





(9) For some function $f(x)$, the calculation of $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ at some $x=a$, gives ___

I. the slope of the tangent line to $f(x)$ at $x=a$.
II. the average rate of change of $f(x)$ from $x=a$ to $x=a+h$.
III. $f\,'(a)$, the derivative of $f(x)$ at $x=a$.
IV. the slope of the secant line from $x=a+h$ to $x=a$.

(a) I and III only
(b) II and IV only
(c) III only
(d) I only
(e) II, III, and IV only

(10) Find the value of k that makes $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x < 3 \\ \sqrt{5kx+16} & x \geq 3 \end{cases}$$

(11) Calculate the following limits using algebraic techniques to justify your answer. If the limit has infinte behavior, use limit notation to describe the behavior.

(a) $\lim_{x\to 2} \sqrt[3]{13x+1}$

(b) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

(c) $\lim_{x\to 4} \dfrac{x^2-x-12}{x^2-16}$

(d) $\lim_{x\to \infty} \dfrac{4-x^7+2x}{-3x+9x^6}$

(e) $\lim_{x\to -\infty} \dfrac{4-x^7+2x}{-3x+9x^6}$

(f) $\lim_{x\to -\infty} \dfrac{\pi^2-3x^4+5x^6}{-3x^6+7e^8+2x^3}$

(12) Given $g(x) = \dfrac{4(x-1)^2(x+3)(x+1)}{(x-1)(x+1)^2(x+3)}$, find the coordinates of any holes and the equations of any vertical asymptotes. Then find $\lim_{x\to 1} g(x)$, $\lim_{x\to -3} g(x)$, and $\lim_{x\to -1} g(x)$.

(13) Find the horizontal asymptotes for $f(x) = \dfrac{4+e^{-2x}+2e^{7x}}{8e^{7x}-7e^{-4x}}$.

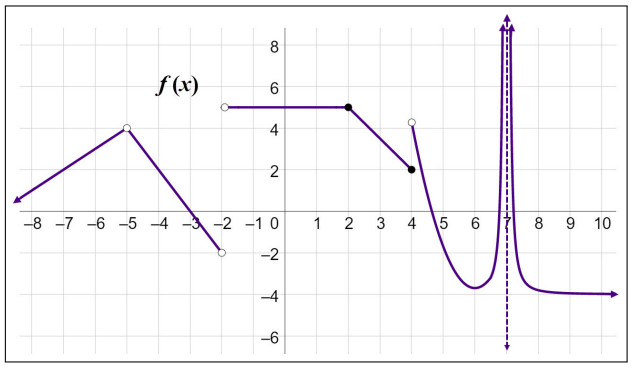
(14) Given $h(x) = \begin{cases} \ln(x^2+4) & x < -1 \\ e^{2x+1} & x > -1 \\ 6 & x = -1 \end{cases}$ find the following limits. Use exact answers and not decimal approximations.

(a) $\lim_{x\to -1^+} h(x)$

(b) $\lim_{x\to -1^-} h(x)$

(c) $\lim_{x\to -1} h(x)$

Use the graph of $f(x)$ below to answer questions #15-17. questions.



(15) The graph of $f(x)$ is not continuous at $x = 4$ because

I. $f(4)$ is not defined. II. $\lim_{x\to 4} f(x)$ does not exist. III. $f(4) \neq \lim_{x\to 4} f(x)$.

(a) I and III only
(b) III only
(c) II and III only
(d) I, II, and III
(e) II only

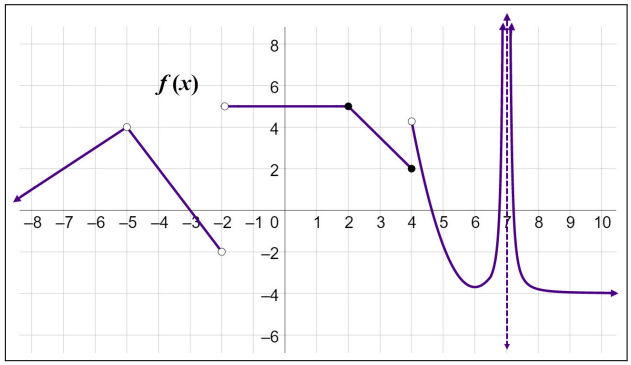
(16) Find $\lim_{x\to -2^+} f(x)$.

(a) 5
(b) -1
(c) -2
(d) 0
(e) DNE

(17) For what values of x does the limit of $f(x)$ not exist?

(a) $x = -5$, -2, 4, and 7
(b) $x = -2$, 2, 4, and 7
(c) $x = -5$, -2, 2, 4, and 7
(d) $x = -2$, 4, and 7
(e) None of these.

(18) Given $g(x) = \dfrac{(x+5)(x-2)}{(x+5)}$, and the graph of $f(x)$ below, find $\lim\limits_{x\to -5}\left(2g(x) + \dfrac{x^2}{\sqrt{f(x)}} - 3\right)$.



(19) Determine the interval(s) where $f(x) = \dfrac{\ln(3x-6)}{\sqrt[6]{-x+5}}$ is continuous.

(a) $(-\infty, 2) \cup (2, 5) \cup (5, \infty)$
(b) $[2, 5]$
(c) $(2, 5) \cup (5, \infty)$
(d) $(2, 5)$
(e) $(-\infty, 2) \cup (2, 5]$

(20) When calculating $\lim\limits_{x\to 4} f(x)$, if when we try to evaluate $f(4)$ we get $\dfrac{0}{0}$, what conclusion can be made about $f(x)$ at $x = 4$?

(a) $f(x)$ must have a hole.
(b) $f(x)$ must have a horizontal tangent line.
(c) $f(x)$ must have a vertical asymptote.
(d) $f(x)$ must have either a hole or a vertical asymptote.
(e) $f(x)$ must have either a hole or a horizontal tangent line.