## Session 5: Sections 2-5 and 2-6 (Implicit Differentiation)

(1) Differentiate each of the following.

(a) $f(x) = \left(5x^4 + 3x^2 + x\right)^{20}$

(b) $g(x) = e^{\sqrt{x}+x^2-5}$

(c) $t(x) = x^2 \ln(2x + e^x)$

(d) $f(x) = e^{e^{2x}} + x^5 2^{x^2+5x} + e^{\pi^2}$

(e) $p(x) = \ln\left(\dfrac{2x^2 + x}{x + 1}\right)$

(f) $f(x) = \dfrac{\log_5(3x^4 + \sqrt{x})}{\sqrt{2x + 3}}$

(g) $f(x) = (\ln x)(x^4 + 2x)^3$

(h) $f(x) = \sqrt{\dfrac{x^2 + 1}{x^2 - 1}}$

(i) $p(x) = \left[\log_2\left(\sqrt{x^2 + e^{3x^2}}\right)\right]^4$

(2) Find the $x-$values where the graph of $f(x) = x^3 e^{-x^2}$ has horizontal tangent lines.

(3) Suppose $F(x) = g(f(x))$ and $f(2) = 3, f'(2) = -3, g(3) = 5$, and $g'(3) = 4$. Find $F'(2)$.

(4) Determine an equation for the line tangent to $f(x) = \ln(3x)$ at $x = 2$.

(5) Use implicit differentiation to find $\dfrac{dy}{dx}$.

(a) $-6e^y + \dfrac{1}{4}y^3 - 2\ln(x) = -8$

(b) $x^2 + x + y^3 - 3y^2 = 45$

(c) $x^2 + xy^3 - 3y^2 = 45$

(d) $3x^4 - \ln\left(y^2 + 36\right) = \dfrac{4}{y^2} - \ln(x)$

(6) Find the slope of the line tangent to the graph of the equation $\dfrac{4x}{2y^3 - 5y^2} = -4$ at $(7, -1)$.

(7) Suppose $x$ and $y$ are functions of $t$ and are related by the equation $\ln\left(3x^2 - 5y\right) + 16 = x^4$. Given $\dfrac{dx}{dt} = 3$, $x = 2$, and $y = \dfrac{11}{5}$, use the information to find $\dfrac{dy}{dt}$.