Session 7: Review for Exam 2

(1) Given $f(x) = \frac{1}{2}e^x\left(x^2 - 4x - 10\right)$, find the intervals of concavity and any inflection points of $f(x)$.

(2) Given the information about $g(x)$ below, find the intervals where $g(x)$ is concave up and concave down. Then find the x-values where $g(x)$ has point(s) of inflection.

- The domain of $g(x)$ is $(-\infty, -2) \cup (-2, \infty)$.
- $g'(x) = \dfrac{(x-7)(x+3)^2}{x+2}$
- $g''(x) = \dfrac{(x-1)(x+3)(2x+1)}{(x+2)^2}$

(3) Given $f'(1) = 0$ and $f''(x) = -6x - 6$, use the second derivative test to determine if there is a local maximum or minimum at $x = 1$.

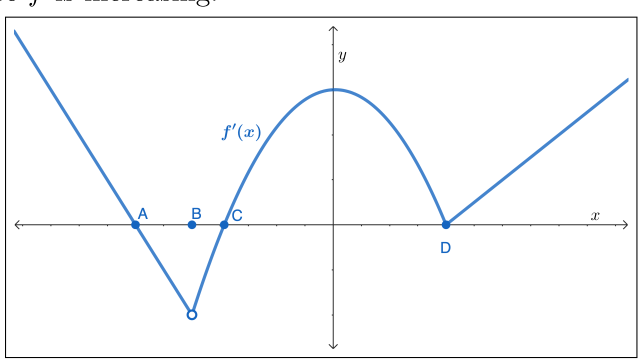
(4) Given the graph of $f\,'$ below and that f is continuous on its domain of $(-\infty, \infty)$, find

(a) the partition numbers of $f\,''$.

(b) the intervals of concavity of f.

(c) the x-values of any inflection points of f.

(d) the intervals where f is increasing.



(5) Sketch a graph of a function that satisfies the following conditions.

- Domian: $(-\infty, \infty)$
- Range: $[2, \infty)$
- Continuous on $(-\infty, \infty)$
- $f'(x) > 0$ on $(-3, 0) \cup (3, \infty)$
- $f'(x) < 0$ on $(-\infty, -3) \cup (0, 3)$
- $f'(0)$ is undefined
- $f''(x) > 0$ on $(-\infty, 0) \cup (0, 6)$
- $f''(x) < 0$ on $(6, \infty)$
- $\lim_{x \to \infty} f(x) = 4$
- $\lim_{x \to -\infty} f(x) = \infty$
- $f(-3) = 2,\ f(0) = 5$

(6) Determine where $f(x) = \frac{3}{5}x^5 - 8x^3$ is decreasing and concave down.

(7) Given $h(x) = e^{-x}\left(-x^2 - 2x - 2\right)$, $h'(x) = x^2 e^{-x}$, and $h''(x) = xe^{-x}(2 - x)$, find all intervals where $h(x)$ is (a) decreasing and concave down, (b) decreasing and concave up (c) increasing and concave down, and (d) increasing and concave up.

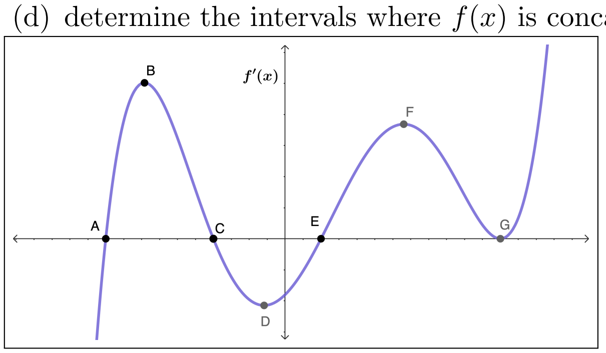
(8) Given the graph of $f'(x)$ below,

(a) find the intervals where $f(x)$ is increasing.

(b) determine the x-values at which $f(x)$ has a local maximum.

(c) determine the x-values at which $f(x)$ has points of inflection.

(d) determine the intervals where $f(x)$ is concave down.



(9) Find the derivatives of the following functions. You do not need to simplify your answers.

(a) $f(x) = (2x+1)\sqrt{x^2+1}$

(b) $f(x) = \dfrac{1}{e^x + e^{-x}}$

(c) $f(x) = e^{2x} \ln\left(2x^3 + x\right)$

(d) $f(x) = 4^{x^4+5}$

(e) $f(x) = \dfrac{\sqrt{x^2+4x-e}}{3^{2x^4+x}}$

(f) $f(x) = (x^2+6x+1)^4$

(g) $f(x) = e^x + \ln x + e^\pi$

(10) Given $f(x) = (x^2+3x)^{2/3}$, find:

(a) the equation of the tangent line to $f(x)$ at $x = -2$.

(b) all critical values for $f(x)$.

(c) all values of x where the line tangent to $f(x)$ is horizontal.

(11) Find the cost of producing the 50^{th} item if a company's cost function is $C(x) = \sqrt{x}(x-10)$ where $C(x)$ is in dollars and x is the number of items produced.

(12) Use implicit differentiation to find the slope of the line tangent to the graph of $19 - 3x^2 + 9^x = 4\sqrt{y}$ at $(0, 25)$.

(13) Find $\dfrac{dy}{dx}$ for the curve $5e^{xy} - 6x^2 = 8y^3 + 7$.

(14) Suppose that $x = x(t)$ and $y = y(t)$ are both functions of t. If $x^2 + y^2 = 40$, and $\dfrac{dx}{dt} = 3$ when $x = 2$ and $y = 6$, what is $\dfrac{dy}{dt}$?

(15) A triangle has a height that is increasing at a rate of 2 cm/second, and its area is increasing at a rate of 4 cm^2/second. Find the rate at which the length of the base of the triangle is changing when the height of the triangle is 4 cm and its area is 20 cm^2.

(16) Given $f(x) = x^3 - 3x + 1$, find (a) the domain of the function, (b) intervals where the function is increasing and decreasing, and (c) all local extrema for the function.