

**Problems:**

1. Sketch the directional field of the following ODEs.

(a) $y' = 2y - 1$

(b) $y' = 2 - y$

(c) $y' = y(y + 3)$

(d) $y' = y(3 - y)$

2. Find the order of the following ODEs. Determine whether each of the ODEs is linear or not.

(a) $t^2 \frac{d^2y}{dt^2} + t \frac{dy}{dt} + 2y = \sin(t)$

(b) $y^2 \frac{d^2y}{dt^2} + t \frac{dy}{dt} + y = e^t$

(c) $y''' + \ln(t)y'' - \sin(t)y = t^2$

(d) $(y'')^3 = \cos(t + y')$

3. Given the ODE

$$t^2y'' + 5ty' + 4y = 0, \quad t > 0.$$

Verify that $y_1(t) = t^{-2}$ and $y_2(t) = t^{-2} \ln(t)$ are solutions of the equation.

4. Determine the value of r for which the following ODE has solutions of the form $y = e^{rt}$:

$$y'' - 5y' + 4y = 0.$$

5. Solve

$$ty' + y = 4t \cos(2t), \quad t > 0.$$

Determine its asymptotic behavior as $t \rightarrow \infty$.