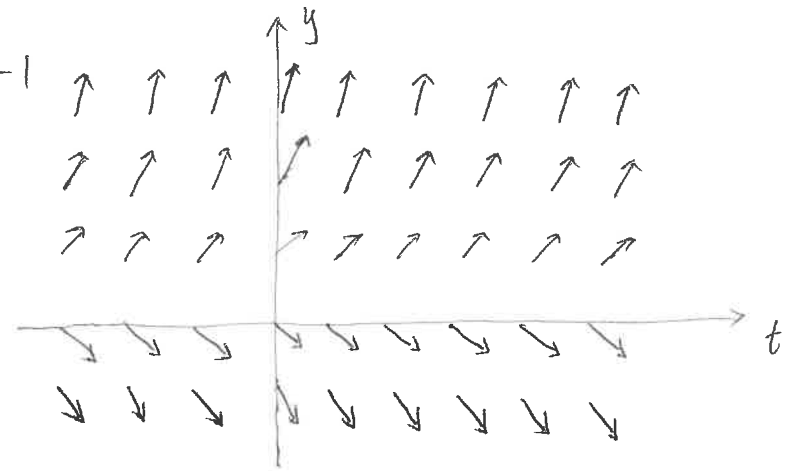
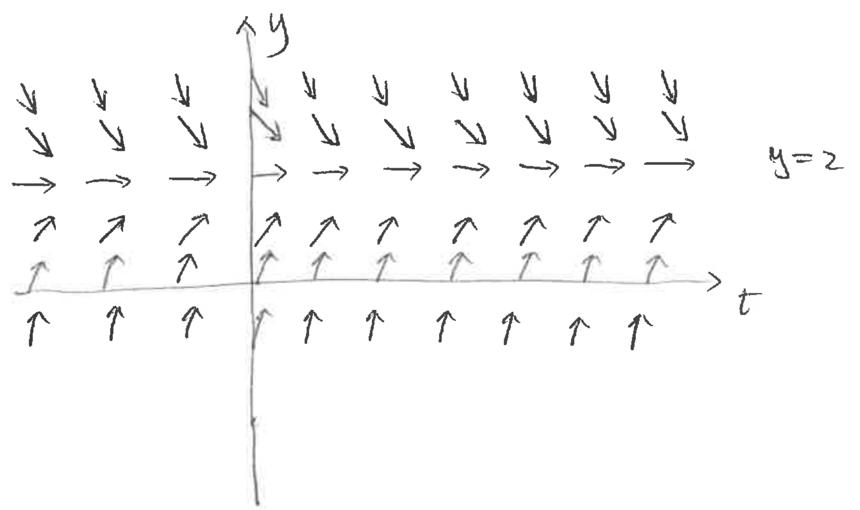
Math 308 (WIR) # 1.

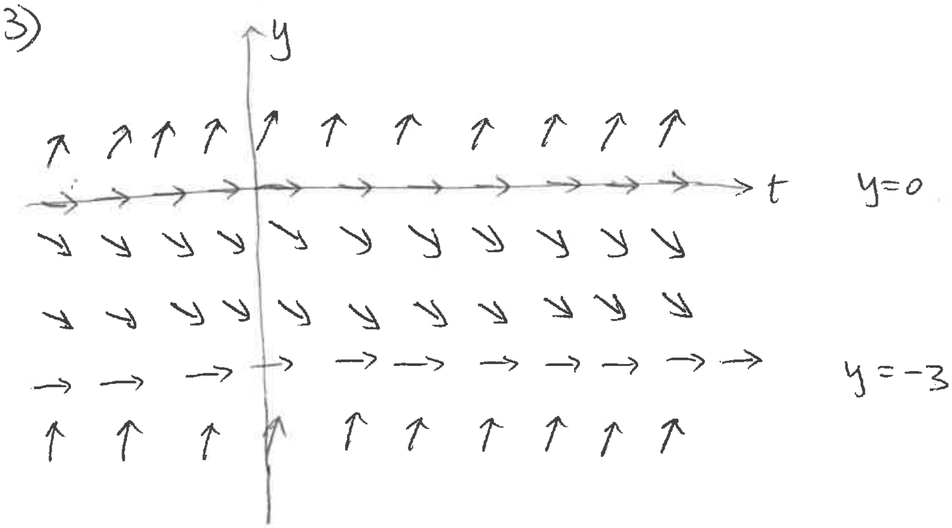
P 1 (a). $y' = 2y - 1$



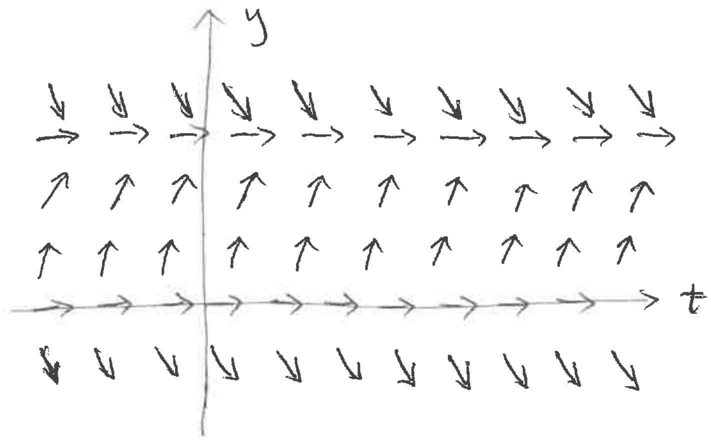
(b) $y' = 2 - y$



(c) $y' = y(y+3)$



(d) $y' = y(3-y)$



$y=3$

$y=0$

P2

(a) $t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin(t)$

2^{nd}-order linear ODE.

(b) $y^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$

2^{nd}-order ODE. But NOT linear

(c) $y''' + \ln(t)\, y'' - \sin(t)\, y = t^2$

3^{rd}-order linear ODE.

(d) $(y'')^3 = \cos(t + y')$.

2^{nd}-order ODE. But NOT linear.

P3.

$$t^2 y'' + 5ty' + 4y = 0. \quad (t > 0).$$

Plug in $y_1(t) = t^{-2}$. Compute $y_1' = -2t^{-3}$ and $y_1'' = 6y^{-4}$.

So $$t^2 \cdot 6y^{-4} + 5t \cdot (-2t^{-3}) + 4 \cdot t^{-2}$$

$$= 6y^{-2} - 10t^{-2} + 4t^{-2}$$

$$= 0 \quad \text{(right-hand side).} \quad \checkmark$$

Then $y_1(t) = t^{-2}$ is a solution.

Plug in $y_2(t) = t^{-2}\ln(t)$: compute $y_2' = -2t^{-3}\ln(t) + t^{-3}$

and $$y_2'' = 6t^{-4}\ln(t) - 2t^{-4} - 3t^{-4}$$

$$= 6t^{-4}\ln(t) - 5t^{-4}$$

So $$t^2\left(6t^{-4}\ln(t) - 5t^{-4}\right) + 5t\left(-2t^{-3}\ln(t) + t^{-3}\right) + 4t^{-2}\ln(t).$$

$$= 6t^{-2}\ln(t) - 5t^{-2} + (-10)t^{-2}\ln(t) + 5t^{-2} + 4t^{-2}\ln(t)$$

$$= 0 \quad \text{(right-hand side)} \quad \checkmark$$

Then $y_2(t) = t^{-2}\ln(t)$ is another solution.

P4. $y'' - 5y' + 4y = 0$

Assume $y = e^{rt}$ and plug in:

compute $y' = re^{rt}$ and $y'' = r^2 e^{rt}$

So $r^2 e^{rt} - 5re^{rt} + 4e^{rt} = 0$.

$\Rightarrow (r^2 - 5r + 4)e^{rt} = 0$.

$\Rightarrow r^2 - 5r + 4 = 0$ (since $e^{rt} \neq 0$).

$\Rightarrow (r-4)(r-1) = 0 \Rightarrow r = 1,\ r = 4$.

So values of r are $r_1 = 1$ and $r_2 = 4$.

P5 $ty' + y = 4t\cos(2t)$, $(t > 0)$

Standard form: $y' + \frac{1}{t}y = 4\cos(2t)$. (divide by t)

integrating factor: $\mu = e^{\int \frac{1}{t}dt} = e^{\ln t} = t$. $(t>0)$

Then $y = \dfrac{\int \mu(t) \cdot 4\cos(2t)\,dt}{\mu(t)} = \dfrac{\int t \cdot 4\cos(2t)\,dt}{t}$.

(integration by parts). $= \dfrac{2\sin(2t)\cdot t - \int 2\sin(2t)\cdot 1\,dt}{t}$

$= 2\sin(2t) + \cos(2t)\cdot\frac{1}{t} + \frac{C}{t}$

Asymptotic behavior as $t \to \infty$:

$y \approx 2\sin(2t)$ oscillating as $t \to \infty$

(since $\lim_{t\to\infty}\left[\cos(2t)\cdot\frac{1}{t} + \frac{C}{t}\right] = 0$).