Week In Review 2

Math 308 - Spring 2023
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Problems:

1. Solve the initial value problem

$$y' + \frac{2}{t}y = \frac{\cos(t)}{t^2}, \qquad y(\pi) = 0, \qquad t > 0.$$

2. Solve the initial value problem

$$y' + \frac{1}{4}y = 3 + 2\cos(2t)e^{-t/4}, \qquad y(0) = 0,$$

and describe its asymptotic behavior as $t \to \infty$.

3. Solve the ODE

$$(x^2 - 9)\frac{dy}{dx} + xy = 0$$

on the suitable interval of validity.

   (a) Assume $x > 3$.
   (b) Assume $y(0) = 1$.

4. Solve $xy' = (1 - y^2)^{1/2}$.

5. Solve the initial value problem

$$y' = \frac{3x^2 - e^x}{2y - 5}, \qquad y(0) = 1.$$

Write your solution in explicit form.

6. Solve the initial value problem

$$\sin(2x)dx + \cos(3y)dy = 0, \qquad y(\pi/2) = \pi/3.$$

Write your solution in explicit form.

7. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water owing in at a rate of 2 L/min, the well-stirred solution owing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

8. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 200° F when freshly poured, and 1 min later has cooled to 190° F in a room at 70° F, determine when the coffee reaches a temperature of 150° F.