Problems:

1. Determine the interval of validity (largest possible interval) of the solution without solving the ODEs.

   (a)
   $$(4-t^2)y' + 2ty = 3t^2, \qquad y(-3) = 1.$$

   (b)
   $$(t-3)y' + (\ln t)y = 2t, \qquad y(1) = 2.$$

2. Given the autonomous equation
   $$\frac{dy}{dt} = (y-4)(y-2)(y+1).$$

   (a) Find the critical (equilibrium) points.
   (b) Sketch the graph of $f(y)$ versus y.
   (c) Draw the phase line.
   (d) Classify equilibrium points (determine the stability).
   (e) Sketch several graphs of solutions in the ty-plane.

3. Given the autonomous equation
   $$\frac{dy}{dt} = (y-3)^2(y-1)(y+2)^2.$$

   (a) Find the critical (equilibrium) points.
   (b) Draw the phase line.
   (c) Classify equilibrium points (determine the stability).
   (d) Sketch several graphs of solutions in the ty-plane.

4. Another equation that has been used to model population growth is the Gompertz equation
   $$\frac{dy}{dt} = ry \ln \frac{K}{y} \qquad (y > 0),$$
   where r and K are positive constants.

   (a) Find the critical (equilibrium) points.
   (b) Classify equilibrium points (determine the stability).
   (c) Solve the Gompertz equation with initial condition $y(0) = y_0$.

5. Determine whether each equation is exact. If it is exact, find the solution.

   (a) $(2x+4y)dx + (2x-2y)dy = 0$
   (b) $(2x+3)dx + (2y-2)dy = 0$

6. Solve the initial value problem and write the solution in explicit form. Determine the interval where the solution is valid.
   $$(2x-y)dx + (2y-x)dy = 0, \qquad y(1) = 3.$$

7. Show that the following equation is not exact but becomes exact when multiplied by the given integrating factor. Then solve the equation.
   $$x^2y^3dx + x(1+y^2)dy = 0, \qquad \mu(x,y) = \frac{1}{xy^3}.$$