Problems:

1. Consider the following 1st order ODE

$$(3xy + y^2)\,dx + (x^2 + xy)\,dy = 0.$$

Verify that it is not an exact equation. Given $M(x, y)\,dx + N(x, y)\,dy = 0$, if

$$\frac{M_y - N_x}{N} = k(x) \qquad \text{only depends on } x,$$

then one can compute an integrating factor by

$$\mu(x) = e^{\int k(x)\,dx}.$$

Use the formula above to find μ, then solve the equation.

2. Solve

$$y'' + 3y' + 2y = 0.$$

3. Solve the initial value problem

$$y'' + 4y' + 3y = 0, \qquad \text{with } y(0) = 2 \text{ and } y'(0) = -1.$$

4. Determine the largest interval in which the given initial value problem is guaranteed to have a unique solution

$$t(t-4)y'' + 3ty' + 4y = 2, \qquad \text{with } y(3) = 0 \text{ and } y'(3) = -1.$$

5. Find the Wronskian of $y_1 = e^{-2t}$ and $y_2 = te^{-2t}$.

6. Given $y_1 = \cos(2t)$ and $y_2 = \sin(2t)$. Do they form a fundamental set of solutions for the following ODE.

$$y'' + 4y = 0$$

If yes, what is the general solution?

7. If the Wronskian of f and g is $W = 3e^{4t}$, and if $f(t) = e^{2t}$, find $g(t)$.

8. Consider the following 2nd order linear ODE

$$t^2y'' + ty' - y = 0.$$

Verify that $y_1(t) = t$ is a solution. Use Abel's formula

$$W(t) = Ce^{-\int P(t)\,dt}$$

$$\text{for} \qquad y'' + P(t)y' + Q(t)y = 0$$

to compute the Wronskian and find another solution $y_2(t)$. Then find the general solution y.

9. Solve

$$y'' - 2y' + 6y = 0.$$

10. Solve the initial value problem

$$y'' - 2y' + 5y = 0, \qquad \text{with } y(\pi/2) = 0 \text{ and } y'(\pi/2) = 2.$$