

**Problems:**

1. Solve the initial value problem

$$y'' - 6y' + 9y = 0, \quad \text{with } y(0) = 0 \text{ and } y'(0) = 2.$$

2. Use the method of reduction of order to find a second solution of the following ODE

$$t^2 y'' + 3ty' + y = 0, \quad (t > 0) \quad \text{given } y_1(t) = t^{-1}.$$

Midterm Review:

3. Solve the initial value problem

$$xy' + (2x - 3)y = 4x^4, \quad y(1) = 0.$$

4. Find the general solution in explicit form.

$$\frac{1}{x} \frac{dy}{dx} = y^2 \sin(x)$$

5. What is the maximum interval on which the solution of the initial value problem exists and is unique by the the existence/uniqueness theorem?

$$(t^2 - 1)y' + t^2 y = t - 3, \quad y(-2) = 2023$$

6. Consider the autonomous equation

$$\frac{dy}{dt} = (y - 2)^2(y + 1)(y + 3).$$

Find the equilibrium points. Classify the equilibrium points (determine the stability). Draw some sample solution curves in the directional field picture.

7. A tank contains 200 L of saline water with the salt concentration 3 g/L. A mixture containing a concentration of 5 g/L of salt enters the tank at a rate of 10 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression for the amount of salt in the tank at time t .

8. Determine whether the following equation is exact. If it is, find the general solution in explicit form.

$$(2xy - \sin(x)) dx + (x^2 + 2y) dy = 0$$

9. Find the general solution of the following ODE

$$y'' - 7y' + 12y = 0.$$

10. Show that the given y_1 and y_2 form a fundamental set of solutions. Find the general solution of the ODE.

$$t^2 y'' - 2y = 0 \quad (t > 0)$$
$$y_1(t) = t^2 \quad \text{and} \quad y_2(t) = t^{-1}$$