Problems:

1. Find the form of the particular solution for the following ODEs.
   (a) $y'' - 3y' + 2y = -2t^2e^{4t}$
   (b) $y'' - 3y' + 2y = -2t^2e^{2t}$
   (c) $y'' - 4y' + 4y = -2t^2e^{4t}$
   (d) $y'' - 4y' + 4y = -2t^2e^{2t}$
   (e) $y'' - 4y' + 4y = 3e^{2t}\cos(2t)$
   (f) $y'' - 4y' + 13y = 3e^{2t}\cos(2t)$
   (g) $y'' - 4y' + 13y = 3e^{2t}\cos(3t)$
   (h) $y'' - 5y' = t^3$

2. Use method of undetermined coefficients to find the general solution.
   (a) $y'' - 2y' - 3y = 3e^{2t}$
   (b) $y'' - 2y' - 3y = -3te^{-t}$
   (c) $y'' - 2y' - 3y = 3e^{2t} - 3te^{-t}$

3. Solve the initial value problem by using the method of undetermined coefficients

$$y'' - 2y' + y = te^t + 4, \qquad y(0) = 1 \text{ and } y'(0) = 1.$$

4. Use method of variation of parameters to find a particular solution of the ODE

$$y'' - y' - 2y = 2e^{-t}.$$

5. Verify that the given y_1 and y_2 form a fundamental set of solutions for the corresponding homogeneous equation. Then find the general solution of the non-homogeneous ODE.

$$ty'' - (t+1)y' + y = t^2e^{2t} \qquad (t > 0)$$

$$y_1(t) = t + 1 \qquad \text{and} \qquad y_2(t) = e^t$$

6. A 2 kg block is hung on a spring which is attached to the ceiling. The spring is stretched 98 cm. After that, the block is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm/s. Assuming downward from equilibrium is the positive direction, and assuming no resistance and external force, find the position function $y(t)$ of the block at any time t. Determine the frequency, period, and amplitude of the motion.

7. A 2 kg block is hung on a spring which is attached to the ceiling. The spring is stretched 98 cm. After that, the block is lifted up 10 cm above its equilibrium position and given an initial upward velocity of 20 cm/s. Meanwhile, the blog is acted on by an external force of $12\sin(2t)$ N, and a damping force is measured to be 1.2 N when the speed of the blog is 10 cm/s. Assuming downward from equilibrium is the positive direction, find the position function $y(t)$ of the block at any time t.