Problems:

1. Find the inverse Laplace transform of the following.

   (a) $F(s) = \dfrac{3}{s^2+4}$

   (b) $F(s) = \dfrac{1-2s}{s^2+4s+5}$

   (c) $F(s) = \dfrac{2s-3}{s^2-4}$

   (d) $F(s) = \dfrac{8s^2-4s+12}{s(s^2+4)}$

2. Use the Laplace transform to solve the initial value problem.

   (a) $y'' + 3y' + 2y = e^{-t}$ with $y(0) = 0$ and $y'(0) = 1$

   (b) $y'' + 9y = 3\sin(2t)$ with $y(0) = 0$ and $y'(0) = 2$

3. Write $f(t)$ into combination of Heaviside functions $u_c(t)$, then find $\mathcal{L}\{f(t)\}$.

   (a) $f(t) = \begin{cases} 0 & 0 \le t < 2 \\ t-2 & 2 \le t < 4 \\ t^2 - 7t + 14 & t \ge 4 \end{cases}$

   (b) $f(t) = \begin{cases} 0 & 0 \le t < 2 \\ t-4 & 2 \le t < 4 \\ t^2 - 9t + 5 & t \ge 4 \end{cases}$

4. Find the inverse Laplace transform of the following.

   (a) $F(s) = \dfrac{e^{-s} + e^{-2s} - e^{-3s}}{s}$

   (b) $F(s) = \dfrac{2}{s^2-4s+13}$

   (c) $F(s) = \dfrac{2(s-1)e^{-2s}}{s^2-4s+13}$

5. Use the Laplace transform to solve the initial value problem.

$$y'' + 2y' + 2y = f(t) \qquad \text{with } y(0) = 0 \text{ and } y'(0) = 1$$

$$\text{where } f(t) = \begin{cases} 0 & 0 \le t < \pi \\ 1 & \pi \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$$

6. Use the Laplace transform to solve the initial value problem.

$$y'' + 4y = \sin(t) - u_{2\pi}(t)\sin(t - 2\pi) \qquad \text{with } y(0) = 0 \text{ and } y'(0) = 0$$