Problems:

1. Compute the following.

   (a) $\int_{-\infty}^{\infty} \delta(t-3)\, dt$

   (b) $\int_{-\infty}^{\infty} \delta_3(t) t^2 \cos(t-2)\, dt$

   (c) $\mathcal{L}\{(3t+1)\delta_3(t)\}$

2. Use the Laplace transform to solve the initial value problem.

   (a) $y'' + y = \delta_{2\pi}(t)\cos(t)$ with $y(0) = 0$ and $y'(0) = 1$

   (b) $y'' + 2y' + 2y = 3\delta_{\pi}(t)$ with $y(0) = 1$ and $y'(0) = 0$

3. Given

$$h(t) = \int_0^t (t-\tau)^2 \cos(2\tau)\, d\tau.$$

   Find $\mathcal{L}\{h(t)\}$.

4. Use convolution theorem to find the inverse Laplace transform of the following.

$$F(s) = \frac{s}{(s+1)(s^2+4)}$$

5. Use the Laplace transform to solve the initial value problem.

$$y'' + 9y = h(t) \qquad \text{with } y(0) = 0 \text{ and } y'(0) = 1$$

$$\text{where} \qquad h(t) = \int_0^t \sin(2(t-\tau))\delta_4(\tau)\, d\tau$$

6. Use the Laplace transform to solve the initial value problem. Your solution will have a term as a convolution integral.

$$y'' + 3y' + 2y = \cos(t^2) \qquad \text{with } y(0) = 1 \text{ and } y'(0) = 0$$

7. Write the following into a single series with general term in x^n.

   (a) $\sum_{n=1}^{\infty} n a_n x^{n-1} + x \sum_{k=0}^{\infty} a_k x^k$

   (b) $x \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n$