

Problem 1:

Topics: continuous random variables, Normal distribution, empirical rule

1. Which of the following would indicate that a dataset is **not** bell-shaped³?
 - a. The range is equal to 5 standard deviations.
 - b. The range is larger than the interquartile range.
 - c. The mean is much smaller than the median.
 - d. There are no outliers.
 - e. None of the above

Problem 2

2. What is the z-score of $x = 5$ if it is 1.8 standard deviations below the mean?
Answer z-score = 1.8

Problem 3:

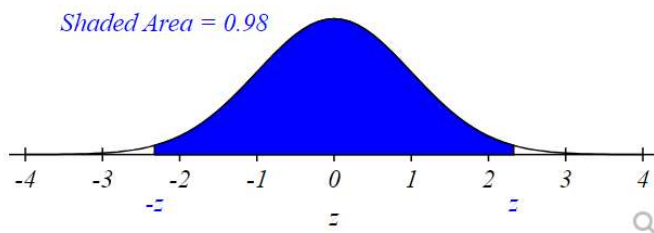
Topics: continuous random variable, standard normal distribution, probability, use of the Z table

What percent of a standard normal distribution $N(\mu = 0, \sigma = 1)$ is found in each region⁴? Be sure to draw a graph

3. $Z < 1.35$ Answer: 91.15%
4. $Z > 1.48$ Answer: 6.94%
5. $0.4 < Z < 1.5$ Answer 27.78%
6. $Z < -20.92$ or $Z > 20.97$ Answer $2x.1151$

Problem 4:

Using the standard normal distribution, find the two z-scores that that form the middle shaded region. The shaded region is symmetric about $z = 0$. Round your z-scores to two decimal places.



Negative z-score = ♂

Positive z-score = ♂

¹ Math-UOttawa ² UVermont ³ Utts ⁴ OpenIntro

Problem 5:

Topics: histogram, Normal approximation to data, Normal probability plot, Q-Q plot

7. Can we approximate poker winnings by a normal distribution? We consider the poker winnings of an individual over 50 days. A histogram and normal probability plot of these data are shown in the following figure⁴:

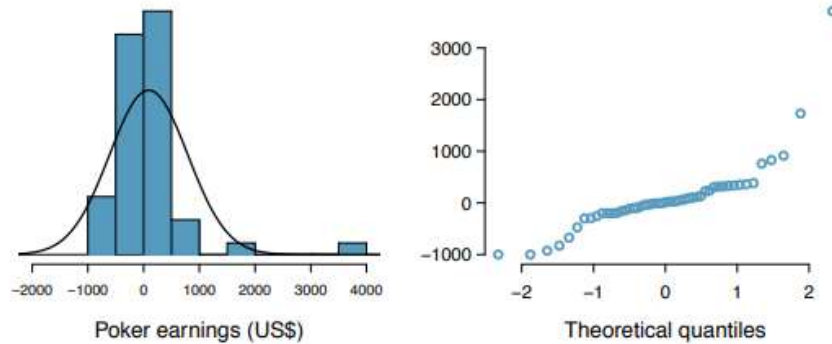


Figure 3.13: A histogram of poker data with the best fitting normal plot and a normal probability plot.

Answer: No, both the histogram and the QQ plot show that the distribution is skewed to the right.

Problem 6

8. Overweight baggage. Suppose weights of the checked baggage of airline passengers follow a nearly normal distribution with mean 45 pounds and standard deviation 3.2 pounds. Most airlines charge a fee for baggage that weigh in excess of 50 pounds⁴. Determine what percent of airline passengers incur this fee.

Answer: 0.0594

Problem 7

The cholesterol content of large chicken eggs is normally distributed with a mean of 200 milligrams and standard deviation 15 milligrams.

9. What is the probability that the cholesterol content of a random egg is less than 205 milligrams?

Answer: 0.6293

10. In sixty-seven percent of the eggs, the cholesterol content is less than a certain value “C”. Find the value of “C”.

- a) 0.33
- b) 206.6**
- c) 210
- d) 0.44
- e) 193.4

¹ Math-UOttawa ² UVermont ³ Utts ⁴ OpenIntro



Problem 8

Topics: Normal distribution, parameters of the normal distribution, z-score, quartiles, use of the Z table

Auto insurance premiums. Suppose a newspaper article states that the distribution of auto insurance premiums for residents of California is approximately normal with a mean of \$1,650. The article also states that 25% of California residents pay more than \$1,800⁴.

11. What is the z-score that corresponds to the top 25% of the standard normal distribution?

Answer: .67

12. What is the mean insurance cost? What is the cutoff for the 75th percentile?

Answer: \$1,800

13. Identify the standard deviation of insurance premiums in LA.

Answer: \$223.88

Problem 9

For each of the following situations, state the parameter of interest. Also identify the corresponding statistic.

14. In a survey, one hundred college students are asked how many hours per week they spend on the Internet.

X: hours per week spend on internet. Continuous r.v.

Parameter of interest: Mean (μ) – “mean number of hours per week spend on internet”

Statistic: \bar{X}

15. In a survey, one hundred college students are asked how many hours per week they spend on the Internet.

Problem 10

16. A sampling distribution is the probability distribution for which one of the following:

- a) A sample
- b) A sample statistic
- c) A population
- d) A population parameter
- e) None of the above

¹ Math-UOttawa 2. UVermont 3 Utts ⁴ OpenIntro



Problem 11

The average number of acres burned by all wildfires in the United States is 780 acres with a standard deviation 500 acres. The distribution of acres burned by wildfires is bell shaped.

17. What is the probability that a random wildfire burns more than 800 acres?

Answer - 0.484

The average number of acres burned by all wildfires in the United States is 780 acres with a standard deviation 500 acres. Of course, some wildfires burn thousands of acres, so the distribution of acres burned by wildfires is strongly right skewed.

A simple random sample of 200 wildfires is to be taken from this population and the sample mean acres burned calculated. Use this to answer the next two questions.

18. What is the probability to have a sample mean that is higher than 800 acres? Find the closest answer.

- a) 0.391
- b) 0.484
- c) 0.516
- d) 0.286
- e) none of above

19. What is the third quartile (Q3) of the sampling distribution of sample mean acres burned? Find the closest answer.

- a) 1116 acres
- b) 756 acres
- c) 804 acres
- d) 782 acres
- e) 815 acres

Problem 12

The distribution of the number of eggs laid by a certain species of hen during their breeding period is 35 eggs with a standard deviation of 18.2. Suppose a group of researchers randomly samples 45 hens of this species, counts the number of eggs laid during their breeding period, and records the sample mean.

20. Define the random variable of interest.

X : number of eggs laid by a certain species of hen.

21. Which are the parameters of the population distribution of X?

$\mu = 35$ and $\sigma = 18.2$



22. Which is the shape of the population distribution of X?

Unknown.

23. Suppose the researchers take all the possible samples of size 45 and estimate the sample mean for each sample. Which is the name of the distribution that they obtain by plotting all the estimated sample means?

Answer: The sampling distribution of \bar{X}

24. Would you expect the shape of this distribution to be symmetric, right skewed, or left skewed? Explain your reasoning.

Symmetric bell shape due to Central Limit Theorem, as $n \geq 30$ and observations are independent.

25. Calculate the variability of the sampling distribution and state the appropriate term used to refer to this value.

$$\text{Standard deviation of } (\bar{X}) = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{18.2}{\sqrt{45}}$$

26. Suppose the researchers' budget is reduced and they are only able to collect random samples of 10 hens. The sample mean of the number of eggs is recorded, and we repeat these 1,000 times, and build a new distribution of sample means. How will the variability of this new distribution compare to the variability of the original distribution?

$$\text{Standard deviation of } (\bar{X}) = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{18.2}{\sqrt{10}}, \text{ the variability will be larger.}$$

Problem 13

Suppose that the mean outstanding credit card balance for all young couples in the US is \$650 with a standard deviation of \$420. The distribution is highly skewed to the right.

27. A simple random sample of 16 young couples is selected and their mean credit card balance is calculated. Therefore, we can use the Central Limit Theorem to estimate the probability that the sample mean is less than \$500?

No, because the population distribution is skewed and the sample size is less than 30.

28. Suppose now that one random sample of size 200 is selected from the population of young couples. The mean of this sample is \$623. Based on this information, fill in the blanks:

The shape of the distributions of this one random sample is skewed to the right with mean \$623 and the shape of the sampling distribution of the sample mean for samples of size 200 is approximately Normal with a mean of \$650.

29. What is the probability that the sample mean of a random sample of size 200 is larger than 680?

$$P(\bar{X} > 680)$$