TEXAS A&M UNIVERSITY Math Learning Center

Problem 1

At a large university, the probability that a student takes calculus and statistics in the same semester is 0.0125. The probability that a student takes statistics is 0.125; the probability that a student takes calculus is 0.3

1. Find the probability that a student is taking calculus, given that he or she is taking statistics.

a) 0.1
b) 0.1125
c) 0.0016
d) 0.1375
e) 0.4800

2. Is the event of taking calculus independent of the event of taking statistics? Justify your answer numerically.

Using the answer in question two, P(C|S)=.1 and it is not equal to P(C)=.3. Therefore, the first condition for independence is not satisfied.

Problem 4

The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English (foreign language) at home, and 4.2% fall into both categories.

Consider P(Below)=P(B)=0.146; P(Foreign)=P(F)=0.207 and $P(B \cap F) = 0.042$

3. Are living below the poverty line and speaking a foreign language at home disjoint?
   No, because there are Americans that live below poverty and speak a foreign language.
   $P(B \cap F) \neq 0$

4. What percent of Americans live below the poverty line and only speak English at home?
   0.146 – 0.042= 0.104 (Hint – use a venn diagram)

5. What percent of Americans live below the poverty line or speak a foreign language at home?
   $P(B \cup F) = P(B) + P(F) - P(B \cap F) = 0.146 + 0.207 - 0.042 = 0.311$

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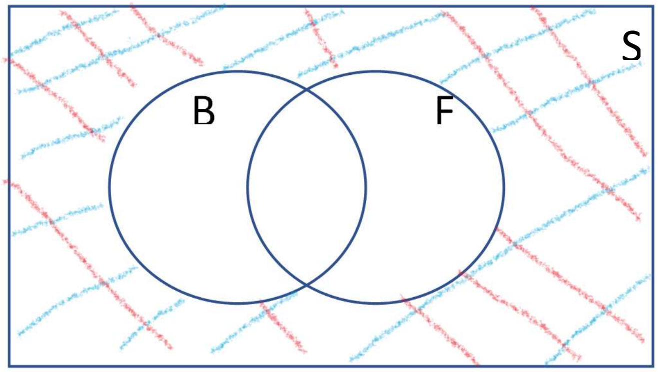
6. What percent of Americans live above the poverty line and only speak English at home? Use a venn diagram.

$$P(B^c \cap F^c) = 1 - P(B \cup F)$$
= 1 - .311 =0.689



7. Is the event that someone lives below the poverty line independent of the event that the person speaks a foreign language at home?

They are not independent because

P(B ∩ F) = 0.042 ≠ P(B) × P(F) = 0.146 x 0.207 = 0.0302

Problem 5

Consider P(A) = 0.3, P(B) = 0.7

8. Can you compute P(A and B) if you only know P(A) and P(B)?
If A and B are independent then $P(A \cap B) = P(A) \times P(B)$

9. Assuming that events A and B arise from independent random processes,

   a. What is P(A and B)?

   $$P(A \cap B) = 0.3 \times 0.7 = .21$$

   b. What is P(A or B)?

   $$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.7 - 0.21 = 0.79$$

   c. What is P(A|B)?

   $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.21}{0.7} = 0.3$$

10. If we are given that P(A and B) = 0.1, are the random variables giving rise to events A and B independent?

A and B are not independent because $P(A \cap B) = 0.1 \neq P(A) \times P(B) = 0.21$

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11. If we are given that P(A and B) = 0.1, what is P(A|B)?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.7} = 0.1428$$

Problem 6

A quiz consists of 3 multiple choice questions with 4 possible answers each. If a student guesses the answer to each question, what is the probability that they get all the answers correct?

$$P(C_1 \text{ and } C_2 \text{ and } C_3) = P(C_1) \times P(C_2) \times P(C_3) = 0.25 \times 0.25 \times 0.25 \times 0.25$$

Problem 7[1]

Topics: contingency table, conditional probability, joint probability, marginal probability.

The family college data set contains a sample of 792 cases with two variables, teen and parents, and is summarized in the following Table:

		parents		
		degree	not	Total
teen	college	231	214	445
	not	49	298	347
	Total	280	512	792

The teen variable is either college or not, where the college label means the teen went to college immediately after high school. The parent's variable takes the value degree if at least one parent of the teenager completed a college degree.

12. If at least one parent of a teenager completed a college degree, what is the chance the teenager attended college right after high school?

P(C|D) = 231/280

13. Probability of a random teenager from the study went to college rightafter highschool:

P(C)= 445/792

Problem 8[1]

Topics: general multiplication rule

A smallpox data set provides a sample of 6,224 individuals from the year 1721 who were exposed to smallpox in Boston. Doctors at the time believed that inoculation, which involves exposing a person to the disease in a controlled form, could reduce the likelihood of death.

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Suppose we are given only two pieces of information: 96.08% of residents were not inoculated, and 85.88% of the residents who were not inoculated ended up surviving.

14. How could we compute the probability that a resident was not inoculated and lived?

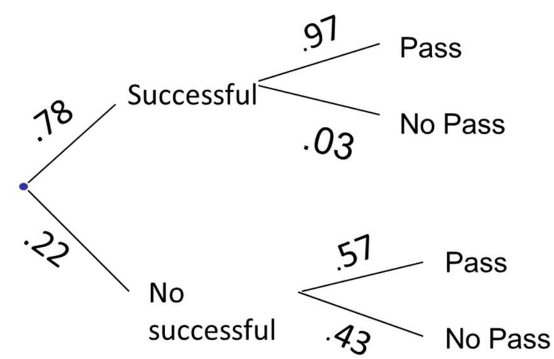
P (Not inoculated and survived) = P (survived | not inoculated) x P (not inoculated)
= .8588 x .9608

Problem 1[1]

Topics: tree diagram, conditional probability, addition rule.

After an introductory statistics course, 78% of students can successfully construct tree diagrams. Of those who can construct tree diagrams, 97% passed, while only 57% of those students who could not construct tree diagrams passed.

15. Organize this information into a tree diagram.



16. What is the probability that a randomly selected student passed?

P(Pass)= P (Successful and Pass) + P (No Successful and Pass) = .78x.97 + .22x.57

17. Compute the probability a student pass if it is known that she couldn't construct a tree diagram?

P (Pass| No successful) =.57

Problem 2[1]

You draw a card from a deck. If you get a red card, you win nothing. If you get a spade, you win $5. For any club, you win $10 plus an additional $20 for the ace of clubs.

18. Construct the probability distribution of the random variable for the amount you win.

X: amount you win

x	$0 (red)	$5 (spade)	$10 (club)	$20 (ace of clubs)
P(X=x)	26/52	13/52	12/52	1/52

19. Find the expected amount you will win if you draw a card

E(X)= $0 (26/52)+$5 (13/52) +$10 (12/52) + $20 (1/52) = $3.94

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20. About how much you expect to win if you play draw a card 20 times

    20 E(X) = 20 ($3.94) = $78.8

Problem 3

Topics: discrete random variable, probability distribution, expected value

The probability distribution for the delay in hours of the evening flight from Chicago to New York is as follows:

x	1	2	3	4	5	6
P(X=x)	0.1	0.1	0.2	0.3	0.2	0.1

21. What is the probability that a randomly selected evening flight from Chicago to New York is delayed more than 3 hours?

a) 0.3
b) 0.1
c) 0.6
d) 0.7
e) None of the above

22. If we take a sample of 100 evening flights from Chicago to New York, how many of them you would expect to have a delay of at least 4 hours?

    a) 10
    b) 20
    c) 30
    d) 60
    e) It cannot be calculated

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