# Math 150 - Week-In-Review 3

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## Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Write the given functions in standard form. Then determine the vertex, whether the vertex is a maximum or minimum, and the axis of symmetry.
   a) $g(x) = -2x^2 - 10x - 11$
   b) $h(x) = \frac{1}{3}x^2 - 2x + 1$

2. Find the x-intercepts of the following functions.
   a) $f(x) = x^2 - 8x + 15$
   b) $h(x) = \frac{1}{3}x^2 - 2x + 1$

3. For the given polynomial functions, determine the end behavior of the graph.
   a) $f(x) = -4x^8 + 7x^5 - 1$
   b) $g(x) = 11x^7 + 6x - 10$
   c) $h(x) = 14x^2 - 9x^3 + 6$

4. Find the zeros and their multiplicities for the following functions, then determine the end behavior and maximum number of turning points.
   a) $k(x) = 2x^3 + 5x^2 + 9x$
   b) $g(x) = -x^3 + 8x^2 - 16x$
   c) $h(x) = 4x^4 + 12x^3 + 9x^2$

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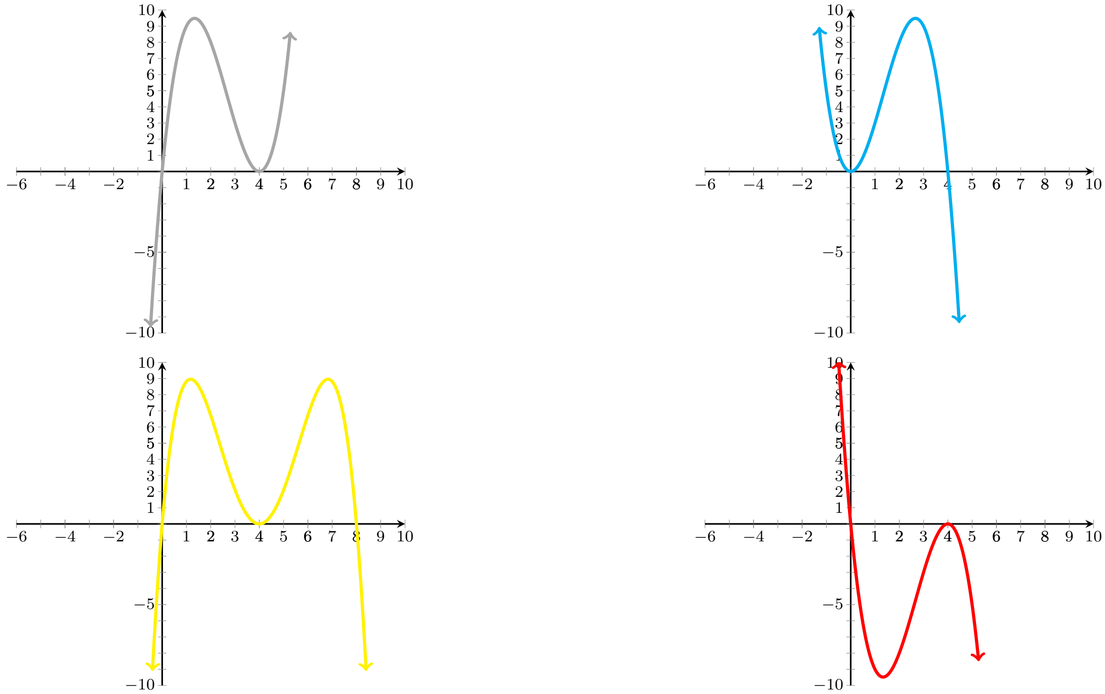
5. Match the function $g(x) = -x^3 + 8x^2 - 16x$ to its graph.



6. Determine the quotient with fractional remainder (if necessary) of the following.

$$(2x^3 - 8x^2 + 3x - 9) \div (x^2 + 1)$$

7. Find the domain, holes, vertical asymptote(s), y-intercept, x-intercept(s), horizontal asymptote, and graph the following functions.

a) $f(x) = \dfrac{2x+5}{x+2}$

b) $g(x) = \dfrac{6x^2 - 17x + 5}{6x^2 - 13x - 5}$

8. Solve $-x^2 + 9x \geq -22$.

9. Solve $(x+5)^3(1-2x)^2 < 0$.

10. Solve $9x^2 + 6x + 1 > 0$.

11. Solve $(x - 100)^2 \leq 0$.

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12. Perform the indicated operation on the functions $f(x) = 3x^2 + 5$ and $g(x) = x + 7$ and determine the domain of each new function.

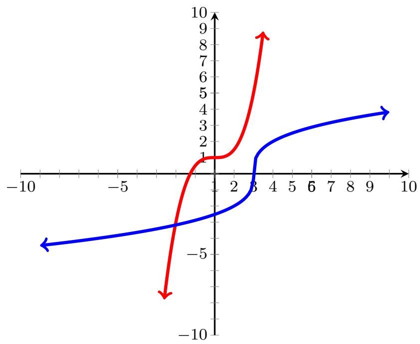
    a. $(f + g)(x)$

    b. $(fg)(x)$

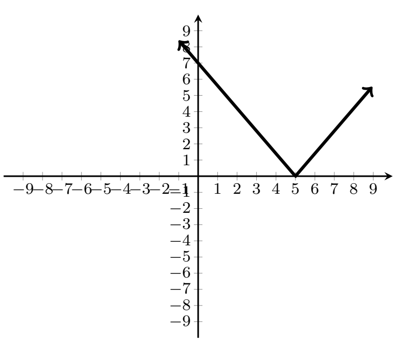
    c. $\left(\dfrac{f}{g}\right)(x)$

    d. $(f \circ g)(x)$

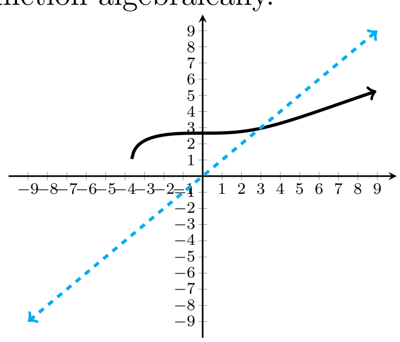
13. Graphically verify whether $f(x) = 2\sqrt[3]{x-2}$ and $g(x) = \dfrac{x^3+2}{2}$ are inverse functions.



14. For the following graph, determine whether the function has an inverse function on its entire domain. If the answer is no, is it possible to restrict the domain so the function is 1-1?



15. Determine whether the function $g(x) = \sqrt[4]{x^3 + 50}$ has an inverse, and, if it does, find the inverse function algebraically.



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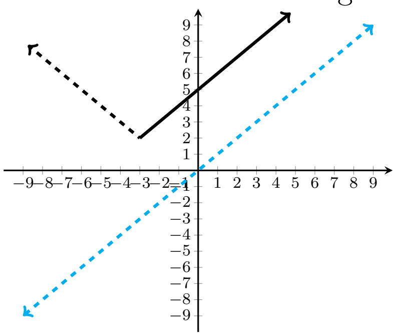
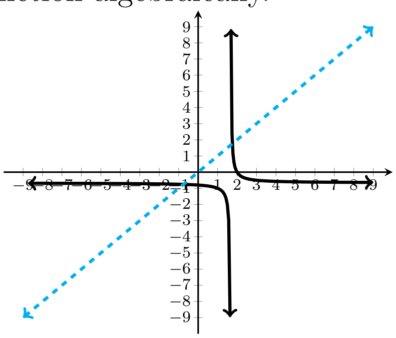
16. Determine whether the function $h(x) = |x + 3| + 2$ where $x \geq -3$ has an inverse, and, if it does, find the inverse function algebraically.



17. Determine whether the function $f(x) = \dfrac{4 - 2x}{3x - 5}$ has an inverse, and, if it does, find the inverse function algebraically.



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# SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. You can also see them all by viewing the Week 3 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Write the given functions in standard form. Then determine the vertex, whether the vertex is a maximum or minimum, and the axis of symmetry.
   a) $g(x) = -2x^2 - 10x - 11$

   $g(x) = -2\left(x + \frac{5}{2}\right)^2 + \frac{3}{2}$, vertex (maximum): $\left(-\frac{5}{2}, \frac{3}{2}\right)$, Axis of symmetry $x = -\frac{5}{2}$

   b) $h(x) = \frac{1}{3}x^2 - 2x + 1$

   $g(x) = \frac{1}{3}(x-3)^2 - 2$, vertex (minimum): $(3, -2)$, Axis of symmetry $x = 3$

2. Find the x-intercepts of the following functions.
   a) $f(x) = x^2 - 8x + 15$

   $(3, 0)$ and $(5, 0)$

   b) $h(x) = \frac{1}{3}x^2 - 2x + 1$

   $(3 + \sqrt{6}, 0)$, $(3 - \sqrt{6}, 0)$

3. For the given polynomial functions, determine the end behavior of the graph.
   a) $f(x) = -4x^8 + 7x^5 - 1$

   As $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to -\infty$

   b) $g(x) = 11x^7 + 6x - 10$

   As $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$

   c) $h(x) = 14x^2 - 9x^3 + 6$

   As $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to -\infty$

4. Find the zeros and their multiplicities for the following functions, then determine the end behavior and maximum number of turning points.
   a) $k(x) = 2x^3 + 5x^2 + 9x$

   $x = 0$ odd multiplicity

   b) $g(x) = -x^3 + 8x^2 - 16x$

   $x = 0$ odd multiplicity, $x = 4$ even multiplicity

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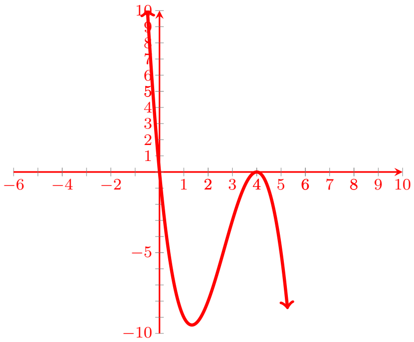
c) $h(x) = 4x^4 + 12x^3 + 9x^2$

$x = 0$ even multiplicity, $x = -\dfrac{3}{2}$ even multiplicity

5. Match the function $g(x) = -x^3 + 8x^2 - 16x$ to its graph.

Based on the characteristics of the graph, the fourth graph matches the $g(x)$.




6. Determine the quotient with fractional remainder (if necessary) of the following.

$$(2x^3 - 8x^2 + 3x - 9) \div (x^2 + 1)$$

$2x - 8 + \dfrac{x - 1}{x^2 + 1}$

7. Find the domain, holes, vertical asymptote(s), y-intercept, x-intercept(s), horizontal asymptote, and graph the following functions.

a) $f(x) = \dfrac{2x + 5}{x + 2}$

Domain: $(-\infty, -2) \cup (-2, \infty)$

Holes: None

VA: $x = -2$

y-int: $\left(0, \dfrac{5}{2}\right)$

x-int: $\left(-\dfrac{5}{2}, 0\right)$

HA: $y = 2$

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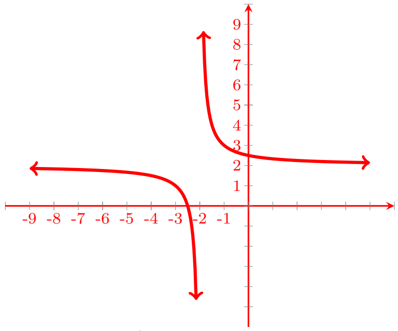



b) $g(x) = \dfrac{6x^2 - 17x + 5}{6x^2 - 13x - 5}$

$g(x) = \dfrac{(3x+1)(2x-5)}{(3x-1)(2x-5)}$

Domain: $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

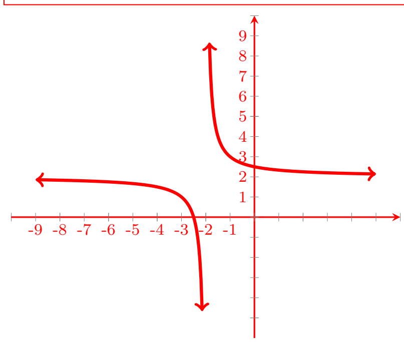
Holes: $\left(\frac{5}{2}, \right)$

VA: $x = -2$

y-int: $\left(0, \frac{5}{2}\right)$

x-int: $\left(-\frac{5}{2}, 0\right)$

HA: $y = 2$




8. Solve $-x^2 + 9x \geq -22$.

$[-2, 11]$

9. Solve $(x+5)^3(1-2x)^2 < 0$.

$(-\infty, -5)$

10. Solve $9x^2 + 6x + 1 > 0$.

$\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$

11. Solve $(x-100)^2 \leq 0$.

$\{100\}$

12. Perform the indicated operation on the functions $f(x) = 3x^2 + 5$ and $g(x) = x + 7$ and determine the domain of each new function.

a. $(f+g)(x)$

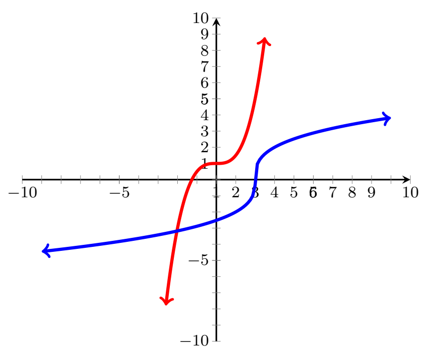
b. $(fg)(x)$

c. $\left(\frac{f}{g}\right)(x)$

d. $(f \circ g)(x)$

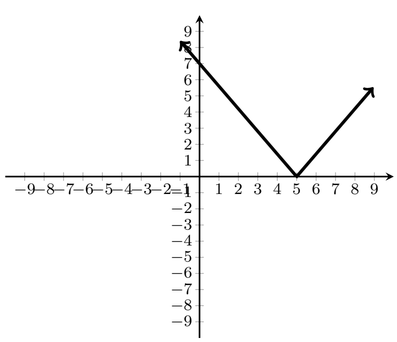
$(f+g)(x) = 3x^2 + x + 12$
$(fg)(x) = 3x^2 + 21x^2 + 5x + 35$
$\left(\frac{f}{g}\right)(x) = \frac{3x^2+5}{x+7}$
$(f(g(x))) = 3x^2 + 42x + 152$

13. Graphically verify whether $f(x) = 2\sqrt[3]{x-2}$ and $g(x) = \frac{x^3+2}{2}$ are inverse functions.



Not inverses because they are not symmetric over $y = x$.

14. For the following graph, determine whether the function has an inverse function on its entire domain. If the answer is no, is it possible to restrict the domain so the function is 1-1?



Not 1-1 as given, but we can restrict our domain to $(-\infty, 5]$ or $[5, \infty)$ and the function will become 1-1.

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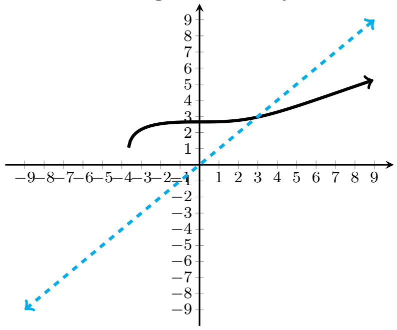
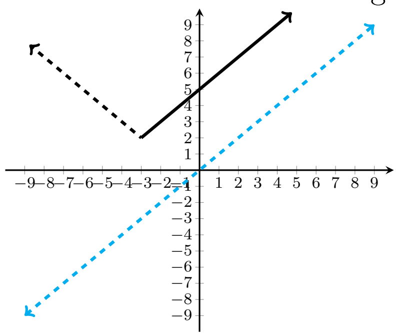
15. Determine whether the function $g(x) = \sqrt[4]{x^3 + 50}$ has an inverse, and, if it does, find the inverse function algebraically.



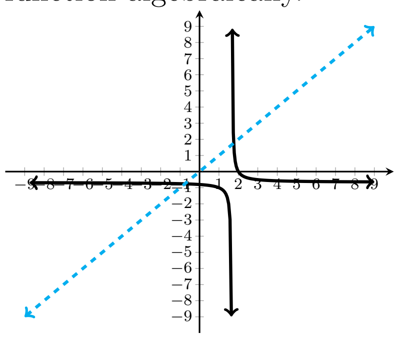
The function does have an inverse. $g^{-1}(x) = \sqrt[3]{x^4 - 50}$

16. Determine whether the function $h(x) = |x + 3| + 2$ where $x \geq -3$ has an inverse, and, if it does, find the inverse function algebraically.



The function has an inverse on the restricted domain. $h^{-1}(x) = x - 5$ with restricted domain $[2, \infty)$.

17. Determine whether the function $f(x) = \dfrac{4 - 2x}{3x - 5}$ has an inverse, and, if it does, find the inverse function algebraically.



The function does have an inverse. $f^{-1}(x) = \dfrac{5x + 4}{3x + 2}$