Math 150 - Week-In-Review 4

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Describe the transformation(s) of the graph of $f(x) = 3^x$ that yield(s) the graph of $g(x) = 3^{-x+3} + 2$.

   Transformations:

   Domain:

   y-intercept:

   Horizontal Asymptote:

2. Describe the transformation(s) of the graph of $f(x) = e^x$ that yield(s) the graph of $g(x) = -2e^{x-5} + 2$.

   Transformations:

   Domain:

   y-intercept:

   Horizontal Asymptote:

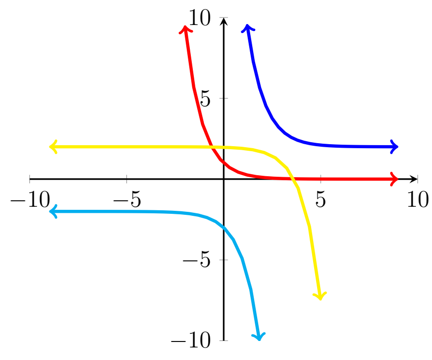
3. Describe the transformation(s) of the graph of $f(x) = 5^x$ that yield(s) the graph of $g(x) = 5^{-0.7x}$, then choose the graph that matches the function.

   Transformations:

   Domain:

   x-intercept(s):

   Horizontal Asymptote(s):



4. Solve the equation for x: $3^{x^2} = 81$

5. Simplify each of the following without a calculator:
   a. $\log_4(64)$
   b. $7^{\log_7(4)} + 2$
   c. $\log(10^{-5})$
   d. $\log_{11}(3x+5) = \log_{11}(9)$

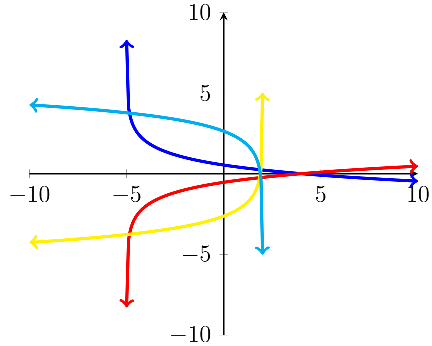
6. Describe the transformations of $f(x) = \log_3(x)$ that yield $g(x) = -\log_3(x+5) + 2$. Then state the domain, x-intercept, and vertical asymptote of the logarithmic function $f(x)$, then choose the graph that matches the function.

   Transformations:

   Domain:

   x-intercept(s):

   Vertical Asymptote(s):



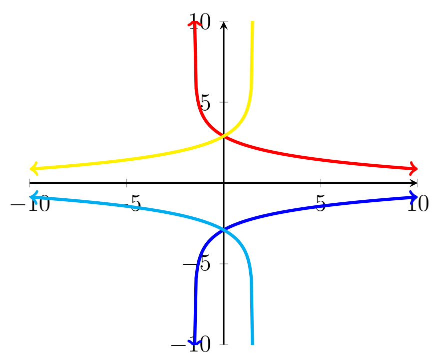
7. Describe the transformations of $f(x) = \ln(x)$ that yield $g(x) = \ln(2x+3) - 4$. Then state the domain, x-intercept, and vertical asymptote of the logarithmic function $g(x)$, then choose the graph that matches the function.

   Transformations:

   Domain:

   x-intercept(s):

   Vertical Asymptote(s):



8. Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)
   (a) $\log_4(64x^2)$
   (b) $\ln \sqrt[3]{\dfrac{x^2}{x^2 - 8x - 20}}$

9. Use the properties of logarithms to condense the expression as a single logarithm. (Assume all variables are positive.)
   (a) $2\log_5(x-1)+4\log_5(y)-1$
   (b) $2\ln(6)-\ln(8)-\ln(81)$

10. Change $\log_7(45)$ to base 5.

11. Change $\log_6(x)$ to base 10

12. Solve each of the following for x. **Always check for extraneous solutions.**
   (a) $e^x = \frac{5}{2}$
   (b) $3^x + 7 = 15$ using the common logarithm
   (c) $\frac{15}{100+e^{2x}} = 3$
   (d) $e^{2x} + 7e^x - 18 = 0$
   (e) $\log_5(4y) = 3$
   (f) $\log_5(x+2) + \log_5(x+3) = \log_5(6)$
   (g) $\log_5(x) + \log_5(x+4) = 1$
   (h) $\log_3(x+2) - \log_3(2x) = 4$

13. The number of bacteria y in a culture after t days is given by the function $y(t) = 100e^{t/8}$.
   (a) What is the initial number of bacteria in the culture?
   (b) How many bacteria are there after 40 days?
   (c) After how many days will there be 4,000 bacteria?

Solutions

Click the boxed answer (also in red) to watch the video solution. You can also see them all by viewing the Week 4 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Describe the transformation(s) of the graph of $f(x) = 3^x$ that yield(s) the graph of $g(x) = 3^{-x+3} + 2$.

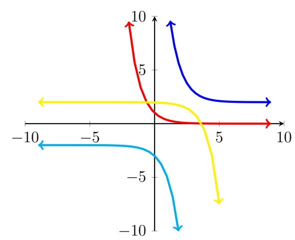
   Transformations: Reflect over y-axis, right 3, up 2.
   OR Left 3, reflect over y-axis, up 2.
   Domain: $(-\infty, \infty)$
   ***y*-intercept:** $(0, 29)$
   Horizontal asymptote: $y = 2$

2. Describe the transformation(s) of the graph of $f(x) = e^x$ that yield(s) the graph of $g(x) = -2e^{x-5} + 2$.

   Transformations: Right 5, reflect over x-axis, vertical stretch by 2, up 2.
   Domain: $(-\infty, \infty)$
   ***y*-intercept:** $(0, -2e^{-5} + 2)$
   Horizontal asymptote: $y = 2$

3. Describe the transformation(s) of the graph of $f(x) = 5^x$ that yield(s) the graph of $g(x) = 5^{-0.7x}$, then choose the graph that matches the function.

   Transformations: Reflect over y-axis, horizontal stretch by 0.7.
   Domain: $(-\infty, \infty)$
   ***y*-intercept:** $(0, 1)$
   Horizontal asymptote: $y = 0$



Red graph

4. Solve the equation for x: $3^{x^2} = 81$

   $x = \pm 2$

5. Simplify each of the following without a calculator:

   a. $\log_4(64) = 3$

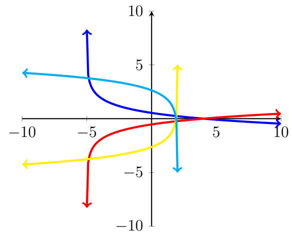
   b. $7^{\log_7(4)} + 2 = 6$

   c. $\log(10^{-5}) = -5$

   d. $\log_{11}(3x + 5) = \log_{11}(9) \implies x = \dfrac{4}{3}$

6. Describe the transformations of $f(x) = \log_3(x)$ that yield $g(x) = -\log_3(x+5)+2$. Then state the domain, x-intercept, and vertical asymptote of the logarithmic function $f(x)$, then choose the graph that matches the function.

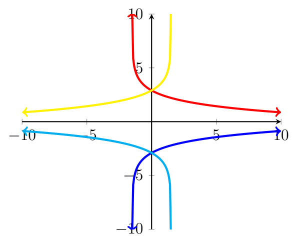
Transformations: Left 5, reflect over x-axis, up 2
Domain: $(-5, \infty)$
y-intercept: $(0, -\log_3(5)+2)$
Vertical asymptote: $x = -5$



Dark blue graph

7. Describe the transformations of $f(x) = \ln(x)$ that yield $g(x) = \ln(2x+3) - 4$. Then state the domain, x-intercept, and vertical asymptote of the logarithmic function $g(x)$, then choose the graph that matches the function.

Transformations: Left 3/2, horizontal shrink by 1/2, down 4
Domain: $(-\frac{3}{2}, \infty)$
y-intercept: $(0, \ln(3) - 4)$
Vertical asymptote: $x = -\frac{3}{2}$



Dark blue graph

8. Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive.)

(a) $\log_4(64x^2)$ $\boxed{3 + 2\log_4(x)}$

(b) $\ln \sqrt[3]{\frac{x^2}{x^2 - 8x - 20}}$ $\boxed{\frac{1}{3}[2\ln(x) - \ln(x-10) - \ln(x+2)]}$

9. Use the properties of logarithms to condense the expression as a single logarithm. (Assume all variables are positive.)

(a) $2\log_5(x-1) + 4\log_5(y) - 1$ $\boxed{\log_5\left(\frac{(x-1)^2 y^4}{5}\right)}$

(b) $2\ln(6) - \ln(8) - \ln(81)$ $\boxed{\ln\left(\frac{1}{18}\right) = -\ln(18)}$

10. Change $\log_7(45)$ to base 5. $\boxed{\frac{\log_5(45)}{\log_5(7)}}$

11. Change $\log_6(x)$ to base 10 $\boxed{\frac{\log(x)}{\log(6)}}$

12. Solve each of the following for x. **Always check for extraneous solutions.**

    (a) $e^x = \frac{5}{2}$ $\boxed{x = \ln\left(\frac{5}{2}\right)}$

    (b) $3^x + 7 = 15$ using the common logarithm $\boxed{x = \frac{\ln 8}{\ln 3}}$

    (c) $\frac{15}{100 + e^{2x}} = 3$ $\boxed{\text{No solution.}}$

    (d) $e^{2x} + 7e^x - 18 = 0$ $\boxed{x = \ln 2}$

    (e) $\log_5(4y) = 3$ $\boxed{x = \frac{125}{4}}$

    (f) $\log_5(x + 2) + \log_5(x + 3) = \log_5(6)$ $\boxed{x = 0}$

    (g) $\log_5(x) + \log_5(x + 4) = 1$ $\boxed{x = 1}$

    (h) $\log_3(x + 2) - \log_3(2x) = 4$ $\boxed{x = \frac{161}{2}}$

13. The number of bacteria y in a culture after t days is given by the function $y(t) = 100e^{t/8}$.

    (a) What is the initial number of bacteria in the culture?

    (b) How many bacteria are there after 40 days?

    (c) After how many days will there be 4,000 bacteria?

    a) 100 bacteria
    b) $100e^5$ bacteria
    c) $8\ln(40)$ days