Math 150 - Week-In-Review 6

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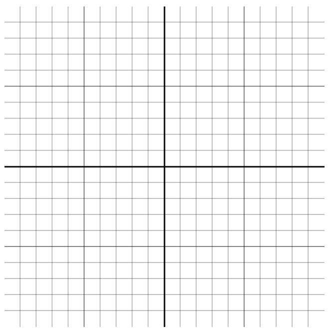
Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Graph $y = 3\tan(3x) - 2$.

   Period: Amplitude:

   Phase Shift: **Period Endpoints** Start: End:



2. Evaluate each:

a. $\arcsin\left(\frac{\sqrt{2}}{2}\right) =$ b. $\sin^{-1}(3) =$ c. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ d. $\arcsin(-1) =$	a. $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ b. $\cos^{-1}\left(\frac{1}{2}\right) =$ c. $\arccos(1) =$ d. $\cos^{-1}(-2) =$
a. $\arctan\left(-\sqrt{3}\right) =$ b. $\tan^{-1}(3) =$ c. $\arctan(-1) =$ d. $\tan^{-1}\left(\frac{5}{2}\right) =$	**Reminders:** • Domain $\arcsin\theta = [-1, 1]$, Range $\arcsin\theta = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ • Domain $\arccos\theta = [-1, 1]$, Range $\arccos\theta = [0, \pi]$ • Domain $\arctan\theta = (-\infty, \infty)$, Range $\arctan\theta = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. Compositions with inverse trig functions
   a. $\cos\left(\arccos\left(-\frac{1}{2}\right)\right)$

   b. $\arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right)$

   c. $\cos(\arccos(2))$

   d. $\arccos\left(\sin\left(\frac{11\pi}{6}\right)\right)$

   e. $\arccos\left(\sin\left(\frac{3\pi}{4}\right)\right)$

   f. $\sin\left(\arctan\left(-\frac{\sqrt{3}}{3}\right)\right)$

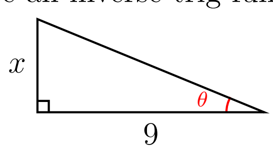
   g. $\tan\left(\arcsin\left(\sqrt{3}\right)\right)$

   h. $\cos\left(\sin^{-1}\left(-\frac{8}{17}\right)\right)$

4. Write an algebraic expression that is equivalent to:
   a. $\tan\left(\sin^{-1}(5x)\right)$

   b. $\csc\left(\cos^{-1}\left(\frac{x}{2}\right)\right)$.

5. Use an inverse trig function to write θ as a function of x.



6. Verify the following identities
   a. $\frac{3\cot^3 t}{\csc t} = 3\cos t(\csc^2 t - 1)$

   b. $\tan x - \cot x = \sec x(2\sin x - \csc x)$

7. Find all solutions for:
   (a) $3\tan x + \sqrt{3} = 0$

   (b) $\sec x - \cos x = 0$

   (c) $\csc^2 x - 2 = 0$

   (d) $2\sin^2 x = 7\cos x + 5$

(e) $2\cos 4x - 1 = 0$

(f) $4\sin^2\left(\frac{x}{2}\right) = 3$

(g) $80\cos\left(\frac{\pi}{3}x + \frac{\pi}{4}\right) - 40\sqrt{2} = 0$

8. Find the exact value of:

   (a) $\sin\left(\frac{5\pi}{12}\right)$, if $\frac{5\pi}{12} = \frac{5\pi}{3} - \frac{5\pi}{4}$

   (b) $\cos(115°)\cos(5°) - \sin(115°)\sin(5°)$

   (c) $\dfrac{\tan(\pi/15) + \tan(4\pi/15)}{1 - \tan(\pi/15)\tan(4\pi/15)}$

9. Rewrite $\sin(x)\cos(3x) + \sin(3x)\cos(x)$ as a single expression.

10. Find all solutions for:

    (a) $\tan(2x) + \tan x = 0$ on $[0, 2\pi)$

    (b) Find all solutions for $4\sin x\cos x = \sqrt{3}$

    (c) Find all solutions for $\dfrac{\cos(2x)}{\cos^2 x} = 1$

11. Find the exact value for $\sin(2\theta)$ and $\cos(2\theta)$, if $\cos(\theta) = -\frac{6}{11}$ and θ is in QII.

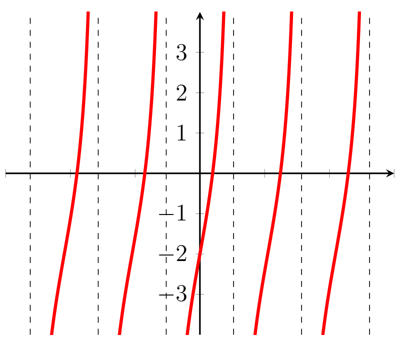
SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. You can also see them all by viewing the Week 6 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Graph $y = 3\tan(3x) - 2$.

Period: $\frac{\pi}{3}$ **Amplitude:** 3

Phase Shift: None **Period Endpoints:** Start: $-\frac{\pi}{6}$ End: $\frac{\pi}{6}$



2. Evaluate each:

a. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$	a. $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$
b. $\sin^{-1}(3) =$ DNE	b. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$
c. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$	c. $\arccos(1) = \frac{\pi}{2}$
d. $\arcsin(-1) = -\frac{\pi}{2}$	d. $\cos^{-1}(-2) =$ DNE

a. $\arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

b. $\tan^{-1}(3)$ OR $\arctan(3)$

c. $\arctan(-1) = -\frac{\pi}{4}$

d. $\tan^{-1}\left(\frac{5}{2}\right)$ OR $\arctan\left(\frac{5}{2}\right)$

Reminders:

- Domain $\arcsin\theta = [-1, 1]$,
  Range $\arcsin\theta = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- Domain $\arccos\theta = [-1, 1]$,
  Range $\arccos\theta = [0, \pi]$
- Domain $\arctan\theta = (-\infty, \infty)$,
  Range $\arctan\theta = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. Compositions with inverse trig functions

a. $\cos\left(\arccos\left(-\frac{1}{2}\right)\right)$ $= -\frac{1}{2}$

b. $\arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right)$ $= -\frac{\pi}{3}$

c. $\cos\left(\arccos\left(2\right)\right)$ $=$ DNE

d. $\arccos\left(\sin\left(\frac{11\pi}{6}\right)\right)$ $= \frac{2\pi}{3}$

e. $\arccos\left(\sin\left(\frac{3\pi}{4}\right)\right)$ $= \frac{\pi}{4}$

f. $\sin\left(\arctan\left(-\frac{\sqrt{3}}{3}\right)\right)$ $= -\frac{1}{2}$

g. $\tan\left(\arcsin\left(\sqrt{3}\right)\right)$ $=$ DNE

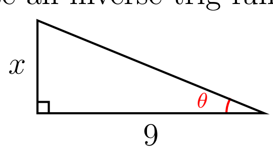
h. $\cos\left(\sin^{-1}\left(-\frac{8}{17}\right)\right)$ $= \frac{15}{17}$

4. Write an algebraic expression that is equivalent to:

a. $\tan\left(\sin^{-1}\left(5x\right)\right)$ $= \frac{5x}{\sqrt{1-25x^2}}$

b. $\csc\left(\cos^{-1}\left(\frac{x}{2}\right)\right)$ $= \frac{2}{\sqrt{4-x^2}}$

5. Use an inverse trig function to write θ as a function of x.



$\theta = \arctan\left(\frac{x}{9}\right)$

6. Verify the following identities

a. $\dfrac{3\cot^3 t}{\csc t} = 3\cos t(\csc^2 t - 1)$

$$\frac{3\cot^3 t}{\csc t}$$
$$= \frac{3\cot t \cdot (\csc^2 t - 1)}{\csc t}$$
$$= \frac{3\dfrac{\cos t}{\sin t} \cdot (\csc^2 t - 1)}{\dfrac{1}{\sin t}}$$
$$= 3\cos t \cdot (\csc^2 t - 1)$$

b. $\tan x - \cot x = \sec x(2\sin x - \csc x)$

$$\tan x - \cot x$$
$$= \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}$$
$$= \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}$$
$$= \frac{\sin^2 x - (1 - \sin^2 x)}{\sin x \cos x}$$
$$= \frac{2\sin^2 x - 1}{\sin x \cos x}$$
$$= \frac{2\sin^2 x - 1}{\sin x} \cdot \frac{1}{\cos x}$$
$$= \left(2\sin x - \frac{1}{\sin x}\right) \cdot \sec x$$
$$= (2\sin x - \csc x) \cdot \sec x$$

7. Find all solutions for:

(a) $3\tan x + \sqrt{3} = 0$

$$x = \frac{5\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$$

(b) $\sec x - \cos x = 0$

$$x = 0 + 2\pi n, \pi + 2\pi n \text{ OR } x = 0 + \pi n$$

(c) $2\sin^2 x = 7\cos x + 5$

$$x = \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n$$

(d) $2\cos 4x - 1 = 0$

$$x = \frac{\pi}{12} + \frac{\pi}{2}n, \frac{5\pi}{12} + \frac{\pi}{2}n$$

(e) $4\sin^2\left(\frac{x}{2}\right) = 3$

$$x = \frac{2\pi}{3} + 4\pi n, \frac{4\pi}{3} + 4\pi n, \frac{8\pi}{3} + 4\pi n, \frac{10\pi}{3} + 4\pi n \text{ OR } \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n$$

(f) $80\cos\left(\frac{\pi}{3}x + \frac{\pi}{4}\right) - 40\sqrt{2} = 0$

$$x = 0 + 6n, \frac{9}{2} + 6n$$

8. Find the exact value of:

(a) $\sin\left(\frac{5\pi}{12}\right)$, if $\frac{5\pi}{12} = \frac{5\pi}{3} - \frac{5\pi}{4}$

$$\frac{\sqrt{6}+\sqrt{2}}{4}$$

(b) $\cos(115°)\cos(5°) - \sin(115°)\sin(5°)$

$$-\frac{1}{2}$$

(c) $\dfrac{\tan(\pi/15) + \tan(4\pi/15)}{1 - \tan(\pi/15)\tan(4\pi/15)}$

$$\sqrt{3}$$

9. Rewrite $\sin(x)\cos(3x) + \sin(3x)\cos(x)$ as a single expression.

$$\sin(4x)$$

10. Find all solutions for:

(a) $\tan(2x) + \tan x = 0$ on $[0, 2\pi)$

$$x = 0 + \pi n, \frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n$$

(b) Find all solutions for $4\sin x\cos x = \sqrt{3}$

$$x = \frac{\pi}{6} + \pi n, \frac{\pi}{3} + \pi n$$

(c) Find all solutions for $\dfrac{\cos(2x)}{\cos^2 x} = 1$

$\boxed{x = 0 + \pi n}$

11. Find the exact value for $\sin(2\theta)$ and $\cos(2\theta)$, if $\cos(\theta) = -\dfrac{6}{11}$ and θ is in QII.

$\boxed{\sin(2\theta) = -\dfrac{12\sqrt{85}}{21}, \cos(2\theta) = -\dfrac{49}{121}}$