Math 150 - Week-In-Review 7

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Determine whether the Law of Sines or the Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $B = 17°$, $C = 135°$ and $c = 65$.

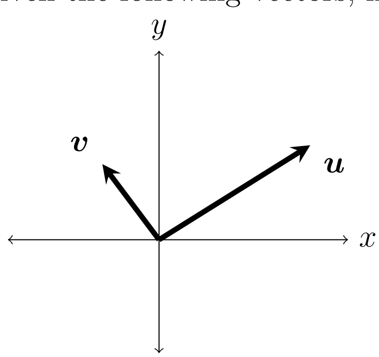
2. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $A = 62°$, $a = 4.2$, $b = 12.4$

3. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $a = 11$, $b = 5$, $c = 13$

4. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $a = 3$, $B = 51°$, $C = 112°$

5. Find the component form and magnitude of the vector $\mathbf{v}$.
   Initial Point: $(2, 4)$ **Terminal Point:** $(-34, -11)$

6. Given the following vectors, find $\overrightarrow{u} + 2\overrightarrow{v}$ and $\overrightarrow{u} - \overrightarrow{v}$.



7. Find a unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$, given that $\mathbf{v} = \langle 3, 7 \rangle$.

8. Find the component form of $\overrightarrow{v} = -\overrightarrow{u} + \overrightarrow{w}$, where $\overrightarrow{u} = 2\overrightarrow{i} - \overrightarrow{j}$, and $\overrightarrow{w} = \overrightarrow{i} + 5\overrightarrow{j}$.

9. Find the component form of v given its magnitude and the angle it makes with the positive x-axis.

   $\|\mathbf{v}\| = 3$, $\theta = 225°$

10. Find $\mathbf{u} \bullet \mathbf{v}$ and $(\mathbf{u} \bullet \mathbf{v})\mathbf{v}$ for $\mathbf{u} = \langle 2, 4\rangle$ and $\mathbf{v} = \langle -6, 2\rangle$.

11. Find the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$ for $\overrightarrow{u} = \langle 2, 4\rangle$ and $\overrightarrow{v} = \langle -6, 2\rangle$.

12. Find the projection of $\mathbf{u} = \langle 2, 4\rangle$ onto $\mathbf{v} = \langle -6, 2\rangle$. Then write $\mathbf{u}$ as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.

13. Compute the difference quotient for $f(x) = \dfrac{x}{2x+1}$.

14. Compute the difference quotient for $g(x) = \sqrt{3x-7}$.

15. Solve using substitution: $\begin{cases} x + 5y = 47 \\ 7x - 8y = -15 \end{cases}$

16. Solve using elimination: $\begin{cases} 4x - 5y = 8 \\ -8x + 10y = -16 \end{cases}$

17. Solve using whichever method you choose: $\begin{cases} 3x - 9y = 11 \\ -4x + 12y = 0 \end{cases}$

Solutions

Click the boxed answer (also in red) to watch the video solution. You can also see them all by viewing the Week 7 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Determine whether the Law of Sines or the Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $B = 17°$, $C = 135°$ and $c = 65$.

$$A = 28° \quad a = \frac{130 \cdot \sin(28°)}{\sqrt{2}} \approx 43.156$$

$$B = 17° \quad b = \frac{130 \cdot \sin(17°)}{\sqrt{2}} \approx 26.876$$

$$C = 135° \quad c = 65$$

2. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $A = 62°$, $a = 4.2$, $b = 12.4$

No solution

3. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.
   $a = 11$, $b = 5$, $c = 13$

$$A = \arccos\left(\frac{7}{13}\right) \quad a = 11$$

$$B = \arccos\left(\frac{265}{286}\right) \quad b = 5$$

$$C = \arccos\left(-\frac{23}{110}\right) \quad c = 13$$

4. Determine whether the Law of Sines or Law of Cosines is needed to solve the triangle, then (if possible), solve the triangle.

   $a = 3$, $B = 51°$, $C = 112°$

   $A = 17°$ $a = 3$

   $B = 51°$ $b = \dfrac{3\sin(51°}{\sin(17°)} \approx 7.974$

   $C = 112°$ $c = \dfrac{3\sin(112°}{\sin(17°)} \approx 9.514$

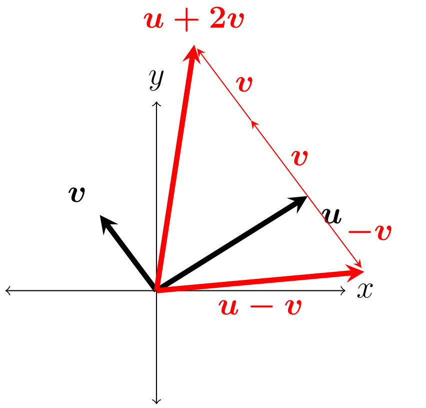
5. Find the component form and magnitude of the vector **v**.

   Initial Point: $(2, 4)$ **Terminal Point:** $(-34, -11)$

   $\vec{v} = \langle -36, -15 \rangle$

   $\|\vec{v}\| = 39$

6. Given the following vectors, find $\vec{u} + 2\vec{v}$ and $\vec{u} - \vec{v}$.



7. Find a unit vector **u** in the direction of **v**, given that $\mathbf{v} = \langle 3, 7 \rangle$.

   $\left\langle \dfrac{3}{\sqrt{58}}, \dfrac{7}{\sqrt{58}} \right\rangle$

8. Find the component form of $\vec{v} = -\vec{u} + \vec{w}$, where $\vec{u} = 2\vec{i} - \vec{j}$, and $\vec{w} = \vec{i} + 5\vec{j}$.

   $\langle -1, 6 \rangle$

9. Find the component form of v given its magnitude and the angle it makes with the positive x-axis.

   $\|\mathbf{v}\| = 3$, $\theta = 225°$

   $\left\langle \dfrac{-3\sqrt{2}}{2}, \dfrac{-3\sqrt{2}}{2} \right\rangle$

10. Find $\mathbf{u} \bullet \mathbf{v}$ and $(\mathbf{u} \bullet \mathbf{v})\mathbf{v}$ for $\mathbf{u} = \langle 2, 4\rangle$ and $\mathbf{v} = \langle -6, 2\rangle$.

$\boxed{\langle 24, -8\rangle}$

11. Find the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$ for $\overrightarrow{u} = \langle 2, 4\rangle$ and $\overrightarrow{v} = \langle -6, 2\rangle$.

$$\boxed{\arccos\left(\frac{-1}{5\sqrt{2}}\right) \approx 98.13^\circ}$$

12. Find the projection of $\mathbf{u} = \langle 2, 4\rangle$ onto $\mathbf{v} = \langle -6, 2\rangle$. Then write $\mathbf{u}$ as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.

$$\text{proj}_{\overrightarrow{v}}\overrightarrow{u} = \left\langle \frac{3}{5}, -\frac{1}{5}\right\rangle$$

$$\langle 2, 4\rangle = \left\langle \frac{3}{5}, -\frac{1}{5}\right\rangle + \left\langle \frac{7}{5}, \frac{21}{5}\right\rangle$$

13. Compute the difference quotient for $f(x) = \dfrac{x}{2x+1}$.

$$\boxed{\text{DQ} = \frac{1}{(2x+1)(2(x+h)+1)}}$$

14. Compute the difference quotient for $g(x) = \sqrt{3x-7}$.

$$\boxed{\text{DQ} = \frac{3}{\sqrt{3x+3h-7}+\sqrt{3x-7}}}$$

15. Solve using substitution: $\begin{cases} x + 5y = 47 \\ 7x - 8y = -15 \end{cases}$

$$\boxed{\left(\frac{301}{43}, \frac{344}{43}\right)}$$

16. Solve using elimination: $\begin{cases} 4x - 5y = 8 \\ -8x + 10y = -16 \end{cases}$

$$\boxed{\left(a, -\frac{8}{5} + \frac{4}{5}a\right)}$$

17. Solve using whichever method you choose: $\begin{cases} 3x - 9y = 11 \\ -4x + 12y = 0 \end{cases}$

$\boxed{\text{No solution.}}$