Note ♯1 (Vectors, Dot Product, Projections)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

[Vectors]

(1) Find a vector $\overrightarrow{AB}$.

(a) $A(-3,4)$, $B(1,-2)$

key: $\langle 4,-6\rangle$

(b) $A(0,0)$, $B(1,1)$

key: $\langle 1,1\rangle$

(c) $A(-2,2)$, $B(-1,3)$

key: $\langle 1,1\rangle$

(2) If $\mathbf{a} = \langle -1, 2 \rangle$ and $\mathbf{b} = \langle 5, 3 \rangle$, find $|\mathbf{a}|$, $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, and $-3\mathbf{a} + 4\mathbf{b}$.

key: $|\mathbf{a}| = \sqrt{5},\ \mathbf{a} + \mathbf{b} = \langle 4, 5 \rangle,\ \mathbf{a} - \mathbf{b} = \langle -6, -1 \rangle,\ -3\mathbf{a} + 4\mathbf{b} = \langle 23, 6 \rangle$

(3) If $|\mathbf{r}| = 2$, and $\mathbf{r}$ makes an angle of 210° with the positive x-axis, find the component of the vector $\mathbf{r}$.

key: $\langle -\sqrt{3}, -1 \rangle$

(4) If $\mathbf{a} = \langle 3, -4 \rangle$, find a vector with length 10 in the direction of $\mathbf{a}$.

key: $\langle 6, -8 \rangle$

[Dot product]

(5) Find $\mathbf{a} \cdot \mathbf{b}$.

(a) $|\mathbf{a}| = 4,\ |\mathbf{b}| = 5$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{3}$

key: $\mathbf{a} \cdot \mathbf{b} = 10$

(b) $\mathbf{a} = \langle -2, -8 \rangle,\ \mathbf{b} = \langle 6, -4 \rangle$

key: $\mathbf{a} \cdot \mathbf{b} = 20$

(c) $\mathbf{a} = \mathbf{i} + \mathbf{j},\ \mathbf{b} = \mathbf{i} - 2\mathbf{j}$

key: $\mathbf{a} \cdot \mathbf{b} = -1$

(6) Find the angle between the vectors.

(a) $\mathbf{a} = \langle 6, 0 \rangle$, $\mathbf{b} = \langle 5, 3 \rangle$

key: $\theta = \arccos\left(\dfrac{5}{\sqrt{34}}\right)$

(b) $\mathbf{a} = \langle 3, 1 \rangle$, $\mathbf{b} = \langle 2, 4 \rangle$

key: $\theta = \arccos\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\pi}{4} = 45°$

(7) Find the values of x such that the given vectors are orthogonal.

(a) $\langle 4, x \rangle$, $\langle x, 1 \rangle$

key: $x = 0$

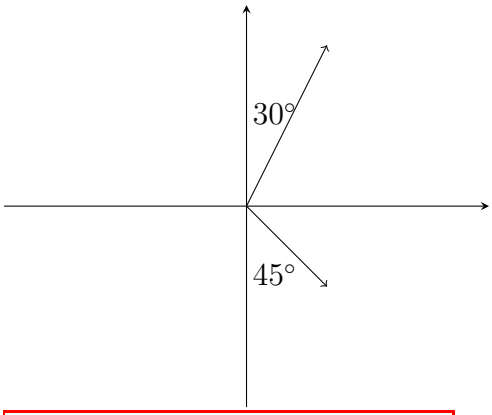
(b) $\langle x, x \rangle$, $\langle 1, x \rangle$

key: $x = 0$ *or* $x = -1$

(8) A force $\mathbf{F} = \langle -3, 4 \rangle$ is used to move an object from the point $(0, 2)$ to the point $(-3, 3)$. How much work is done by the force if distance is measured in meters and force is measured in Newtons?

key: $13\ Nm\ or\ J$

(9) A boat heads in the direction $N30°E$ with a speed of $40mph$. The water current is flowing $S45°E$ with a speed of $6mph$. Find the true speed and direction of the boat.



key: True Course: $\langle 3\sqrt{2}, -3\sqrt{2} \rangle$

key: Speed: $\sqrt{(20+3\sqrt{2})^2 + (20\sqrt{3}-3\sqrt{2})^2}$

key: Direction: N $\theta°E$ *where* $\theta = \arctan\left(\dfrac{20+3\sqrt{2}}{20\sqrt{3}-3\sqrt{2}}\right)$

[Projections]

(10) Find the vector and scalar projection of $\langle 4, 8\rangle$ onto $\langle 2, 1\rangle$.

key: scalar projection: $\dfrac{16}{\sqrt{5}}$, *vector projection:* $\left\langle \dfrac{32}{5}, \dfrac{16}{5} \right\rangle$

(11) Find the vector and scalar projection of $\langle 2, 1\rangle$ onto $\langle 4, 8\rangle$.

key: scalar projection: $\dfrac{4}{\sqrt{5}}$, *vector projection:* $\left\langle \dfrac{4}{5}, \dfrac{8}{5} \right\rangle$

(12) Find the distance from the point $P(2,1)$ to the line $y = 3x + 1$.

key: $\dfrac{6}{\sqrt{10}}$