Note ♯2 (Vector Functions and Parametric Curves, Inverse Trigonometric Functions, The Limit of a Function, Calculating Limits Using the Limit Laws, Continuity, Limits at Infinity)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

[Vector Functions and Parametric Curves]

(1) Eliminate the parameter to find the Cartesian equation of the curve. Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

(a) $x = 2t - 1$, $y = 2 - t$, $-2 \leq t \leq 2$.

key: $y = -\frac{1}{2}x + \frac{3}{2}$

(b) $x = 2t - 1$, $y = t^2 - 1$.

key: $y = \frac{1}{4}(x+1)^2 - 1$

(c) $x = 3\cos\theta,\ y = 4\sin\theta,\ 0 \le \theta \le 2\pi$.

key: $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

(2) Find a vector equation for the line passing through $(1, 3)$ and $(-2, 7)$.

key: $\langle 1 - 3t, 3 + 4t\rangle$

[Inverse Trigonometric Functions]

(3) Find the exact value of the expression.

(a) $\sin\left(\arccos\frac{4}{5}\right)$

key: $\frac{3}{5}$

(b) $\sin\left(2\sin^{-1}\frac{3}{5}\right)$

key: $\frac{4}{5}$

(4) Simplify each expression.

(a) $\tan(\sin^{-1} x)$

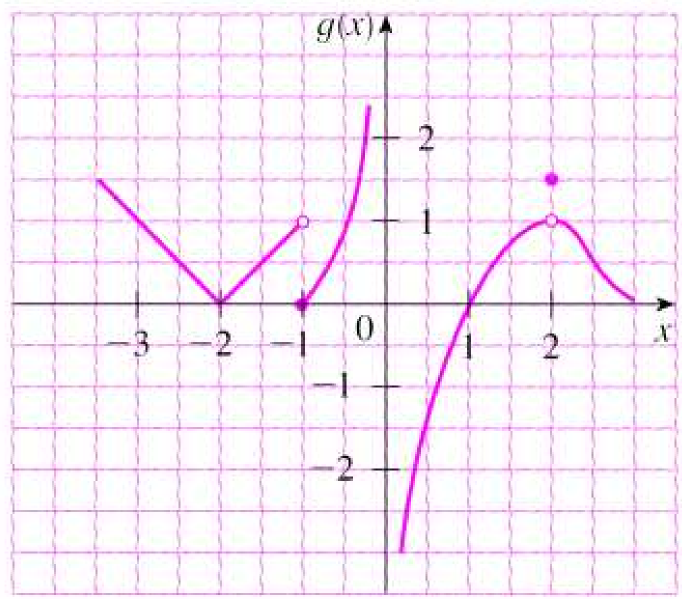
key: $\dfrac{x}{\sqrt{1-x^2}}$

(b) $\sin(\arctan x)$

key: $\dfrac{x}{\sqrt{x^2+1}}$

[The Limit of a Function]

(5) State the value of the given quantity, if it exists, from the given graph.



a. $\lim_{x\to 0^-} g(x)$

b. $\lim_{x\to 0^+} g(x)$

c. $\lim_{x\to 0} g(x)$

d. $\lim_{x\to 2^-} g(x)$

e. $\lim_{x\to 2^+} g(x)$

f. $\lim_{x\to 2} g(x)$

g. $g(2)$

h. $\lim_{x\to -1^-} g(x)$

i. $\lim_{x\to -1^+} g(x)$

j. $\lim_{x\to -1} g(x)$

key: a.$+\infty$, b.$-\infty$, c.DNE, d.1, e.1, f.1, g.1.5, h.1, i.0, j.DNE

(6) Find the limit.

(a) $\lim_{x \to 3} \frac{1}{(x-3)^8}$

key: $+\infty$

(b) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)}$

key: $-\infty$

(c) $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)}$

key: $-\infty$

(d) $\lim_{x \to -2^-} \frac{x-1}{x^2(x+2)}$

key: $+\infty$

(e) $\lim_{x \to -2} \frac{x-1}{x^2(x+2)}$

key:DNE

[**Calculating Limits Using the Limit Laws**]

(7) Evaluate the limit.

(a) $\lim_{x \to -3} \dfrac{x^2 - x - 12}{x + 3}$

key: -7

(b) $\lim_{x \to 1} \dfrac{x^2 - x - 2}{x + 1}$

key: -1

(c) $\lim_{t \to 0} \dfrac{\sqrt{2 - t} - \sqrt{2}}{t}$

key: $-\dfrac{1}{2\sqrt{2}}$

(d) $\lim_{h\to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

key: $-\dfrac{1}{9}$

(e) $\lim_{t\to 1} \left\langle 2t-3,\ \dfrac{t^2-t}{t-1} \right\rangle$

key: $\langle -1, 1\rangle$

(8) Find the limit.

(a) $\displaystyle\lim_{x\to -4^-} \frac{|x+4|}{x+4}$

key: -1

(b) $\displaystyle\lim_{x\to -4^+} \frac{|x+4|}{x+4}$

key: 1

(c) $\displaystyle\lim_{x\to -4} \frac{|x+4|}{x+4}$

key:DNE

(9) Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

evaluate each of the following limits if it exists.

(a) $\lim_{x \to 0^+} f(x)$

key:0

(b) $\lim_{x \to 0^-} f(x)$

key:0

(c) $\lim_{x \to 0} f(x)$

key:0

(d) $\lim_{x \to 1} f(x)$

key:1

(e) $\lim_{x \to 2^-} f(x)$

key:4

(f) $\lim_{x \to 2^+} f(x)$

key:6

(g) $\lim_{x \to 2} f(x)$

key:DNE

[**Continuity**]

(10) Explain why the function $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$ is discontinuous at $x = 0$.

key: $f(x)$ *is NOT continuous at* $x = 0$, *because* $f(x) \neq \lim_{x \to 0} f(x)$.

(11) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$$

key: $a = \dfrac{1}{2}$, $b = \dfrac{1}{2}$

[Limits at Infinity]

(12) Find the limit.

(a) $\lim_{x\to\infty} \dfrac{1-x^2}{x^3-x+1}$

key: 0

(b) $\lim_{x\to\infty} \dfrac{4x^3+6x^2-2}{2x^3-4x+5}$

key: 2

(c) $\lim_{x\to\infty} (\sqrt{9x^2+x}-3x)$

key: $\dfrac{1}{6}$

(d) $\lim_{x\to-\infty} \dfrac{1+x^6}{x^4+1}$

key: $+\infty$

(e) $\lim_{x\to-\infty} \dfrac{\sqrt{1+4x^6}}{2-x^3}$

key: 2

(f) $\lim_{x\to\infty} \dfrac{\sqrt{x+3x^2}}{4x-1}$

key: $\dfrac{\sqrt{3}}{4}$

(g) $\lim_{x\to\infty} \dfrac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$

key: 1

(h) $\lim_{x\to-\infty} \dfrac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$

key: -1