



NOTE #3 (EXAM1 REVIEW)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, [Video List](#).

- (1) Find the scalar and vector projections of $\langle -3, 1 \rangle$ onto $\langle 2, 5 \rangle$.

key: scalar projection: $-\frac{1}{\sqrt{29}}$, vector projection: $\left\langle -\frac{2}{29}, -\frac{5}{29} \right\rangle$

- (2) Vector \mathbf{a} starts at the point $(-3, 1)$ and ends at the point $(6, 3)$. Find a unit vector of \mathbf{a} .

key: $\left\langle \frac{9}{\sqrt{85}}, \frac{2}{\sqrt{85}} \right\rangle$

- (3) Determine a vector equation of the straight line which passes through the point $(1, -1)$ and is parallel to the vector $\langle -2, 3 \rangle$.

key: $\langle 1 - 2t, -1 + 3t \rangle$



- (4) Consider the curve $x(t) = t - 2$, $y(t) = t^2 - 3$. (a) Is the point $(4, 40)$ on the graph of the curve? (b) Eliminate the parameter to find a cartesian equation. (c) Sketch the curve.

key: (a) No, (b) $y = (x + 2)^2 - 3$

- (5) Find a Cartesian equation for the parametric curve given by the equation $x = \frac{\cos t}{2}$, $y = \sin t$, $0 \leq t \leq 2\pi$. And describe the direction of motion as t increases.

key: $4x^2 + y^2 = 1$



- (6) Find the angle $\angle ABC$ of the triangle with the vertices, $A(3, 0)$, $B(0, 3)$, $C(5, 4)$.

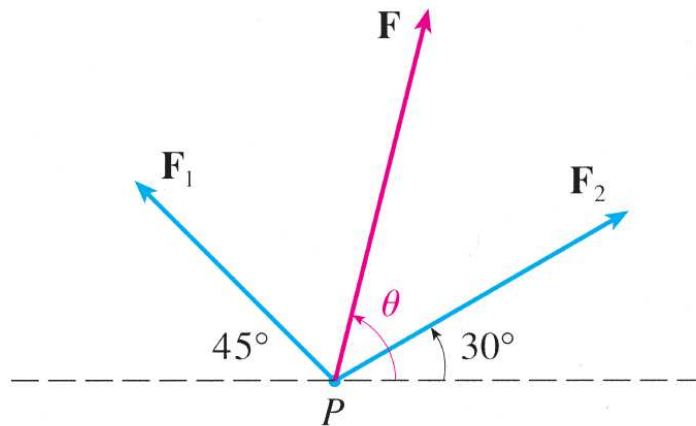
$$\text{key: } \theta = \cos^{-1} \left(\frac{2}{\sqrt{13}} \right)$$

- (7) Find the distance from the point $(1, 2)$ to the line $y = 3x - 4$.

$$\text{key: } \frac{3}{\sqrt{10}}$$



- (8) Two forces \mathbf{F}_1 and \mathbf{F}_2 with magnitudes 10 lb and 12 lb act on an object at a point P as shown in the figure. Find the resultant force \mathbf{F} acting at P as well as its magnitude and its direction.



key: magnitude: $\sqrt{(-5\sqrt{2} + 6\sqrt{3})^2 + (5\sqrt{2} + 6)^2}$, direction: $\theta = \tan^{-1} \left(\frac{5\sqrt{2} + 6}{-5\sqrt{2} + 6\sqrt{3}} \right)$



- (9) A constant force with the vector representation $\mathbf{F} = \langle 10, 18 \rangle$ moves an object along a straight line from the point $(2, 3)$ to the point $(4, 9)$. Find the work done, if the distance is measured in meters and the magnitude of the force is measured in newtons.

key: 128 J

- (10) A woman exerts a horizontal force of 65 lb on a crate as she pushes it up a ramp that is 20 ft long and inclined at an angle of 30° above the horizontal. Find the work done on the box.

key: $650\sqrt{3} \text{ ft} \cdot \text{lb}$

- (11) Find the equation of the line perpendicular to the vector $\langle 3, 5 \rangle$ and passing through the point $(5, 1)$.

key: $\mathbf{r}(t) = \langle 5 - 5t, 1 + 3t \rangle$



- (12) Consider the line $x = 8 - 2t$, $y = 14 + 7t$. (a) Find a vector perpendicular to the line, (b) Find the Cartesian form and (c) sketch the graph.

key: vector perpendicular: $\langle -7, -2 \rangle$, $\langle 7, 2 \rangle$, Cartesian form: $y = -\frac{7}{2}x + 42$

- (13) Simplify.

(a) $\tan(\arccos(\frac{1}{4}))$

key: $\sqrt{15}$

(b) $\sin(\arctan(2))$

key: $\frac{2}{\sqrt{5}}$

(c) $\tan(\arcsin(3x))$

key: $\frac{3x}{\sqrt{1-9x^2}}$



(14) Evaluate the limit.

(a) $\lim_{x \rightarrow 1} \frac{x^2 - x - 2}{x + 1}$

key: -1

(b) $\lim_{t \rightarrow 0} \frac{\sqrt{2-t} - \sqrt{2}}{t}$

key: $-\frac{1}{2\sqrt{2}}$

(c) $\lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$

key: $-\frac{1}{9}$



(d) $\lim_{x \rightarrow -4^-} \frac{|x+4|}{x+4}$
key: -1

(e) $\lim_{x \rightarrow -1^-} \frac{x-2}{x+1}$
key: ∞

(f) $\lim_{x \rightarrow -\infty} \frac{-x^3 + 2x^2 - 4x}{8 + 4x^2 - 5x^3}$
key: $\frac{1}{5}$

(g) $\lim_{x \rightarrow \infty} \frac{e^x + 2e^{-x}}{2e^x - e^{-x}}$
key: $\frac{1}{2}$

(h) $\lim_{x \rightarrow \infty} \ln \left(\frac{x}{x^2 - 3x} \right)$
key: $-\infty$



(i) $\lim_{x \rightarrow \infty} \arctan(e^x)$

key: $\frac{\pi}{2}$

(j) $\lim_{x \rightarrow \infty} \arctan(\ln x)$

key: $\frac{\pi}{2}$

(k) $\lim_{x \rightarrow 0^+} \arctan(\ln x)$

key: $-\frac{\pi}{2}$



(15) Given $-2x - 2 \leq f(x) \leq \frac{1}{2}x^2$, compute $\lim_{x \rightarrow -2} f(x)$.

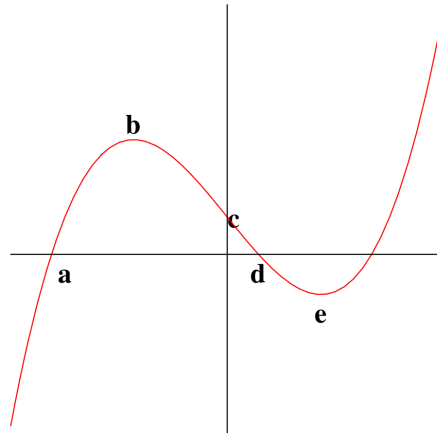
key: 2

(16) Find the vertical and horizontal asymptotes of $f(x) = \frac{(x-1)(x+3)}{x^2-1}$.

key: V.A: $x = -1$, H.A: $y = 1$



(17) Given the graph of $f(x)$ below, sketch the graph of the derivative.



key: See the video

(18) Given $f(x) = \begin{cases} x - 4a & \text{if } x < -2 \\ ax^2 & \text{if } x \geq -2 \end{cases}$. Find the value of a which makes the function continuous everywhere.

key: $a = -\frac{1}{4}$



(19) Which of following intervals must contain a solution to the equation $2x^3 + 16x + 3 = 22$?

- (a) $[-2, -1]$
- (b) $[-1, 0]$
- (c) $[0, 1]$
- (d) $[1, 2]$
- (e) $[2, 3]$

key: d

(20) Find the average rate of change of $f(x) = x^2 + 6$ from $x = -3$ to $x = 1$.

key: -2



(21) Find $f'(x)$ using the limit definition of the derivative for $f(x) = 3x^2 - 4$.

key: $f'(x) = 6x$



(22) Find $f'(x)$ using the limit definition of the derivative for $f(x) = \sqrt{3x + 1}$.

$$\text{key: } f'(x) = \frac{3}{2\sqrt{3x + 1}}$$



(23) Find $f'(x)$ using the limit definition of the derivative for $f(x) = \frac{-2}{x+2}$.

$$\text{key: } f'(x) = \frac{2}{(x+2)^2}$$