Note ♯4 (Derivatives of Polynomial and Exponential Functions, The Product and Quotient Rules, Derivatives of Trigonometric Functions, and Chain Rule)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

(1) Differentiate the functions.

(a) $f(x) = \frac{7}{4}x^4 - 3x^2 + 12$

key: $7x^3 - 6x$

(b) $H(u) = (3u - 1)(u + 5)$

key: $3u^2 + 14u - 5$

(c) $F(r) = \frac{8}{r^3}$

key: $-24r^{-4}$

(d) $y = \frac{\sqrt[3]{x} + x}{x^2}$

key: $-\frac{5}{3}x^{-\frac{8}{3}} - x^{-2}$

(e) $k(x) = e^x + x^e$

key: $e^x + ex^{e-1}$

(f) $f(x) = (3x^2 - x)e^x$

key: $(6x-1)e^x + (3x^2 - x)e^x$

(g) $y = \dfrac{e^x}{4 - e^x}$

key: $\dfrac{e^x(4-e^x) - e^x(-e^x)}{(4-e^x)^2}$

(h) $G(x) = \dfrac{x^2 - 3}{5x + 1}$

key: $\dfrac{(2x)(5x+1) - (x^2-3)(5)}{(5x+1)^2}$

(i) $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 2y^3)$

key: $(-2y^{-3} + 12y^{-5})(y + 2y^3) + (y^{-2} - 3y^{-4})(1 + 6y^2)$

(j) $V(t) = \frac{7+t}{te^t}$

key: $\frac{(1)(te^t) - (7+t)(e^t + te^t)}{(te^t)^2}$

(k) $F(x) = (1 + x + x^2)^{200}$

key: $200(1 + x + x^2)^{199}(1 + 2x)$

(l) $g(\theta) = \cos^2\theta$

key: $-2\cos\theta\sin\theta$

(m) $y = e^{\tan\theta}$

key: $e^{\tan\theta}\sec^2\theta$

(n) $F(t) = e^{t\sin 2t}$

key: $e^{t\sin(2t)}(\sin(2t) + 2t\cos(2t))$

(o) $f(t) = \tan(\sec(\cos t))$

key: $\sec^2(\sec(\cos t))\sec(\cos t)\tan(\cos t)(-\sin t)$

(2) Find $f'(x)$ and $f''(x)$.

(a) $f(x) = (x^3 + 1)e^x$

key: $f'(x) = (x^3 + 3x^2 + 1)e^x,\ f''(x) = (3x^2 + 6x)e^x + (x^3 + 3x^2 + 1)e^x$

(b) $y = e^{e^x}$

key: $y' = e^{e^x} e^x,\ y'' = e^{e^x + x}(e^x + 1)$

(3) Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

key: 120

(4) If $g(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, find $g'(0)$.

key: 96

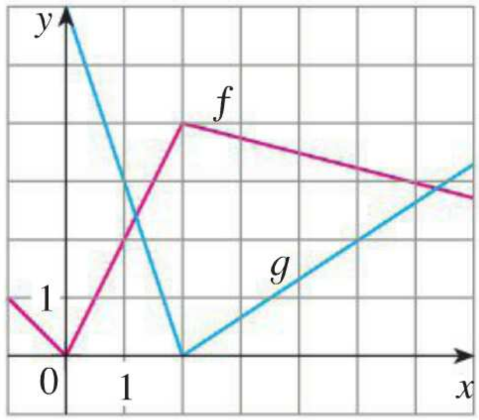
(5) Find the 2020th derivative of $y = \cos 2x$.

key: $y^{(2020)} = 2^{2020}\cos(2x)$

(6) Find the 2020th derivative of $f(x) = xe^{-x}$.

key: $f^{(2020)} = -(2020 - x)e^{-x}$

(7) If f and g are the functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$. Find $u'(1)$, $v'(1)$, and $w'(1)$.



key: $u'(1) = \frac{3}{4},\ v'(1) = DNE,\ w'(1) = -2$