Note ♯5 (Implicit Differentiation, Derivatives of Logarithmic Functions, Derivatives of Vector Functions, Slopes and Tangents to Parametric Curves)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

(1) Find $\dfrac{dy}{dx}$.

(a) $x^3 - xy^2 + y^3 = 1$

key: $\dfrac{dy}{dx} = \dfrac{-3x^2 + y^2}{-2xy + 3y^2}$

(b) $\cos(xy) = 1 + \sin y$

key: $\dfrac{dy}{dx} = \dfrac{-y\sin(xy)}{\cos y + x\sin(xy)}$

(c) $e^y \sin x = x + xy$

key: $\dfrac{dy}{dx} = \dfrac{1 + y - e^y \cos x}{e^y \sin x - x}$

(d) $x \sin y + y \sin x = 1$

key: $\dfrac{dy}{dx} = \dfrac{-\sin y - y \cos x}{x \cos y + \sin x}$

(2) If $g(x) + x\sin(g(x)) = x^2$, find $g'(x)$.

key: $g'(x) = \dfrac{2x - \sin(g(x))}{1 + x\cos(g(x))}$

(3) Find an equation of the tangent line to the curve at the given point.

(a) $x^2 + 2xy + 4y^2 = 12$, $(2, 1)$

key: $y = -\frac{1}{2}x + 2$

(b) $y \sin 2x = x \cos 2y$, $(\pi/2, \pi/4)$

key: $y = \frac{1}{2}x$

(4) Find the derivative of the function.

(a) $y = (\tan^{-1} x)^2$

key: $\dfrac{dy}{dx} = 2(\tan^{-1} x)\dfrac{1}{1+x^2}$

(b) $y = \tan^{-1}(x^2)$

key: $\dfrac{dy}{dx} = \dfrac{2x}{1+x^4}$

(c) $R(t) = \arcsin(1/t)$

key: $\dfrac{dR}{dt} = \dfrac{-1}{t^2\sqrt{1-\frac{1}{t^2}}}$

(d) $f(x) = \arctan(x^2 - x)$

key: $\dfrac{df}{dx} = \dfrac{2x-1}{1+(x^2-x)^2}$

(e) $f(x) = \ln(\sin^2 x)$

key: $f'(x) = \dfrac{2\cos x}{\sin x}$

(f) $g(x) = \ln(xe^{-2x})$

key: $g'(x) = \dfrac{1-2x}{x}$

(g) $f(x) = \log(1+\cos x)$

key: $f'(x) = \dfrac{-\sin x}{(1+\cos x)\ln 10}$

(h) $F(s) = \ln \ln s$

key: $F'(s) = \dfrac{1}{s \ln s}$

(i) $y = \log_2(x \log_5 x)$

key: $y' = \dfrac{\log_5 x + \frac{1}{\ln 5}}{(x \log_5 x)\ln 2}$

(5) If $f(x) = \cos(\ln x^2)$, find $f'(1)$.

key: $f'(1) = 0$

(6) Find an equation of tangent line to the curve $y = x^2 \ln x$ at the point $(1, 0)$.

key: $y = x - 1$

(7) Use the logarithmic differentiation to find the derivative of the function.

(a) $y = \dfrac{e^{-x}\cos^2 x}{x^2+x+1}$

key: $y' = \dfrac{e^{-x}\cos^2 x}{x^2+x+1}\left(-1 - \dfrac{2\sin x}{\cos x} - \dfrac{2x+1}{x^2+x+1}\right)$

(b) $y = x^x$

key: $y' = x^x(\ln x + 1)$

(c) $y = (\ln x)^{\cos x}$

key: $y' = (\ln x)^{\cos x}\left(-\sin x \cdot \ln(\ln x) + \dfrac{\cos x}{x \ln x}\right)$

(8) Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

(a) $r(t) = \langle 2t, t^3 + 1 \rangle$

key: Watch the video

(b) $r(t) = \langle \sin t, \cos t \rangle$

key: Watch the video

(9) Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

(a) $x = 1 - t^3$, $y = t^2 - 3t + 1$; $t = 1$

key: $y = \frac{1}{3}x - 1$

(b) $x = 2\sin\theta$, $y = 3\cos\theta$; $\theta = \pi/4$

key: $y = -\frac{3}{2}x + 3\sqrt{2}$

(10) Find the point(s) on the curve $x = t(t^2 - 3)$, $y = 3(t^2 - 3)$ where the tangent is horizontal or vertical.

key: Horizontal: $(0, -9)$, *Vertical:* $(-2, -6)$, $(2, -6)$