Note ♯6 (Exam2 Review)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

(1) Find the derivative.

(a) $f(x) = e^{x^2}$

key: $f'(x) = e^{x^2} \cdot 2x$

(b) $f(x) = x \sin^7(\cos(6x))$

key: $f'(x) = \sin^7(\cos(6x)) + x \cdot 7\sin^6(\cos(6x)) \cdot \cos(\cos(6x)) \cdot (-\sin(6x)) \cdot 6$

(c) $f(x) = \dfrac{(x-1)^2}{e^{x^2+2x}}$

key: $f'(x) = \dfrac{2(x-1) \cdot e^{x^2+2x} - (x-1)^2 \cdot e^{x^2+2x} \cdot (2x+2)}{e^{2x^2+4x}}$

(d) $f(x) = x\sec^4(5x)$

key: $f'(x) = \sec^4(5x) + x \cdot 4\sec^3(5x) \cdot \sec(5x) \cdot \tan(5x) \cdot 5$

(e) $f(x) = \cos(x + e^{3x})$

key: $f'(x) = -\sin(x + e^{3x}) \cdot (1 + 3e^{3x})$

(f) $f(x) = \dfrac{(4-x)^2}{\tan x}$

key: $f'(x) = \dfrac{2(4-x)\cdot(-1)\cdot\tan x - (4-x)^2 \cdot \sec^2 x}{\tan^2 x}$

(g) $f(x) = \ln(\sin^2 x)$

$$\textit{key: } f'(x) = \frac{2\sin x \cdot \cos x}{\sin^2 x} = \frac{2\cos x}{\sin x}$$

(h) $g(x) = \ln(xe^{-2x})$

$$\textit{key: } f'(x) = \frac{e^{-2x} + x \cdot e^{-2x}(-2)}{x \cdot e^{-2x}}$$

(i) $f(x) = \log_5(1 + \cos x)$

$$\textit{key: } f'(x) = \frac{-\sin x}{(1+\cos x)\ln 5}$$

(j) $f(x) = \arcsin(1/x)$

key: $f'(x) = \dfrac{1}{\sqrt{1-(\frac{1}{x})^2}} \cdot \dfrac{-1}{x^2}$

(k) $f(x) = \sqrt{1-x^2}\arcsin x$

key: $f'(x) = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) \cdot \arcsin x + 1$

(l) $f(x) = \arctan(x^2 - x)$

key: $f'(x) = \dfrac{1}{1+(x^2-x)^2} \cdot (2x-1)$

(2) Find $\dfrac{dy}{dx}$.

(a) $x^2y^3 - 5x^3 = \sec(4y) + 10^{y^2}$

key: $\dfrac{dy}{dx} = \dfrac{2xy^3 - 15x^2}{4\sec(4y)\tan(4y) + 2y \cdot 10^{y^2} \ln 10 - 3x^2y^2}$

(b) $\tan(xy^2) + \sin y = 6x^2 + 8y + 2$

key: $\dfrac{dy}{dx} = \dfrac{12x - y^2 \cdot \sec^2(xy^2)}{2xy \cdot \sec^2(xy^2) + \cos y - 8}$

(3) If $f(x) = \cos(\ln x^2)$, find $f'(1)$.

key: 0

(4) Use the logarithmic differentiation to find the derivative of the function.

(a) $y = x^{\cos x}$

key: $\dfrac{dy}{dx} = x^{\cos x}\left(-\sin x \ln x + \dfrac{\cos x}{x}\right)$

(b) $y = (\ln x)^{\cos(x^2+3)}$

key: $y' = (\ln x)^{\cos(x^2+3)}\left(-2x\sin(x^2+3)\cdot\ln(\ln x) + \dfrac{\cos(x^2+3)}{x\ln x}\right)$

(5) Given that $h(5) = 3$, $h'(5) = -2$, $g(5) = -3$ and $g'(5) = 6$, find $f'(5)$ for each of the following.

(a) $f(x) = g(x)h(x)$

key: 24

(b) $f(x) = \dfrac{g(x)}{h(x)}$

key: $\dfrac{4}{3}$

(c) $f(x) = g(h(x))$

key: Not enough information

(6) Find $h''(1)$ if $h(x) = e^{-x^2}$.

key: $\dfrac{2}{e}$

(7) The vector function $\mathbf{r}(t) = (t + e^t)\mathbf{i} - 2\sin(t)\mathbf{j}$ represents the position of a particle at time t. Find the speed of the object at the point $(1, 0)$.

key: $2\sqrt{2}$

(8) At what point on the graph of $f(x) = \sqrt{x}$ is the tangent line parallel to the line $2x - 3y = 4$?

key: $\left(\frac{9}{16}, \frac{3}{4}\right)$

(9) For what values of x does the curve $y = x + \frac{1}{3}\cos(3x)$ has a horizontal tangent? $(0 \leq x \leq 2\pi)$

key: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

(10) The 77th derivative of $g(x) = \sin(2x)$.

key: $2^{77}\cos(2x)$

(11) Use a linear approximation at $x = 1$ to approximate the value of $\sqrt{1.1}$.

key: $\frac{21}{20}$

(12) A particle moves according to the equation $s(t) = t^2 - t$, where t is measured in seconds and s is in feet. What is the total distance the particle travels during the first 2 seconds?

key: 2.5

(13) Find the point(s) on the curve $x = t^2 + 4t$, $y = t^2 + 2t$ where the tangent line is vertical or horizontal.

key: horizontal at $(-3, -1)$, *vertical at* $(-4, 0)$

(14) Find the equation of the tangent line to the curve parametrized by $x = 5t - t^3$, $y = t^2 - 2t$ at the point corresponding to $t = 0$.

key: $y = -\frac{2}{5}x$

(15) Find the equation of the tangent line to the curve $2x^2y - 3y^2 = -11$ at the point $(2, -1)$.

key: $y = \frac{4}{7}x - \frac{15}{7}$

(16) Consider $f(x) = \begin{cases} ax^2 + x + 1 & \text{if } x \le -1 \\ bx - 1 & \text{if } x > -1 \end{cases}$, find the value of a and b that make $f(x)$ differentiable everywhere.

key: $a = 2, \; b = -3$

(17) Suppose the linear approximation for a function $f(x)$ at $a = 2$ is given by the tangent line $y = -3x + 11$. If $g(x) = (f(x))^2$, find the linear approximation for $g(x)$ at $a = 2$.

key: $L(x) = 25 - 30(x - 2)$